THE SEPTEMBER MEETING OF THE SAN FRANCISCO SECTION.

The sixteenth regular meeting of the San Francisco Section of the American Mathematical Society was held at the University of California, Saturday, September 25, 1909. The following members were present:

Professor R. E. Allardice, Professor H. F. Blichfeldt, Professor M. W. Haskell, Professor L. M. Hoskins, Professor D. N. Lehmer, Professor A. O. Leuschner, Professor H. C. Moreno, Professor C. A. Noble, Mr. E. W. Pounzer, Mr. H. W. Stager, Professor A. L. Whitney.

The following officers were elected for the ensuing year: Professor Blichfeldt, Chairman; Professor Noble, Secretary; Professors Hoskins, Lehmer, Noble, Program Committee.

The next two meetings are to be held respectively at Stanford University February 26, 1910, and at the University of California September 24, 1910.

The following papers were read at this meeting:


4. Professor L. M. Hoskins: “The strain of a gravitating compressible elastic sphere.”

5. Professor A. O. Leuschner: “An equation giving the geocentric distance in the problem of determining parabolic orbits from geocentric observations.”

Mr. McEwen was introduced by Professor Blichfeldt.

Abstracts of the papers are given below in order as numbered in the foregoing list:

1. Professor Lehmer gives a study of the arithmetical properties of the forms included in the pencil $\alpha A + \beta B$, where $\alpha$ and $\beta$ are integers and $A$ and $B$ binary quadratic forms. The
forms of the pencil must have determinants representable by a
certain binary quadratic form \( H = Dx^2 + \Theta xy + D'y^2 \), where
\( D \) and \( D' \) are the determinants of \( A \) and \( B \) and \( \Theta \) is the joint
invariant. If \( A = (abc), B = (a'b'c') \), the form \( J = (ab' - a'b)x^2 +
(\omega - a'c)xy + (bc' - b'c)y^2 \) is also of fundamental importance
in the theory. A pencil may be found having a given form \( J \).
A pencil may or may not be found having a given form \( H \),
according as \( H \) is or is not of the principal genus. The form
\( H \) is the duplicate of \( J \) if \( J \) is a primitive form.

2. By elementary algebraic processes, involving the approxi­
mate evaluation of certain factorials, Professor Blichfeldt
proved that the arithmetical progressions \( k, 11 + k, 22 + k, 
33 + k, \ldots \) (\( k \) prime to \( 11 \)) contain an infinite number of
primes each.

3. Mr. McEwen's paper is in abstract as follows: A viscous
fluid is confined between two parallel planes, one being fixed,
the other having a displacement in its own plane,
\[ x_1 = ae^{-at} \sin \sigma t. \]

Assuming the distance between the planes to be great, the
displacement of the fluid at the distance \( y \) from the moving
plane is
\[ x_2 = ae^{-at - \beta y} \sin (\sigma t - \beta y), \]
where \( \beta = \sqrt{\sigma \rho / 2 \mu}, \rho = \text{density of the fluid}, \mu = \text{the coefficient}
of viscosity of the fluid.

A gravity pendulum is hung so that a small plane attached
to its lower end is parallel to the plane of vibration of the
pendulum and the fluid in which it is immersed. \( x_3 \) is the dis­
placement of this plane. \( x'_1, x'_2, \) and \( x'_3 \) are the maximum
values of \( x_1, x_2, \) and \( x_3 \).
\[ \frac{x'_2}{x'_1} = e^{-\beta (1 + \frac{v}{a})y} \quad \text{and} \quad \frac{x'_3}{x'_1} = \left(1 - \frac{\alpha^2}{2\sigma^2}\right) e^{-\beta (1 + \frac{v}{a})y}, \]
if \( x_3 = ae^{-at} \sin \sigma t \) when the plane is not in the fluid and if
\( \alpha, \sigma \) is small.

4. In Kelvin's well-known solution of the problem of the
strain of an elastic sphere, the bodily forces are assumed to be
known functions of the coordinates of position. When self-
gravitation is considered this solution is inapplicable, except in
the case of incompressibility, because the force of attraction
acting upon any volume element depends in part upon the
change of density produced in that element by the strain and
upon the change of density distribution of the attracting mass.
A solution of the problem taking account of the actual gravi-
tational forces in the strained configuration is given in Professor
Hoskins's paper. The problem is worked out completely for
the case in which the strain is due to disturbing forces of the
type of tidal or centrifugal forces, and numerical results have
been obtained corresponding to several different values of the
ratio of the elastic constants. The strain at any distance from
the center being specified by two quantities — the ellipticity of
the originally spherical surface and the angular displacement of
a radius vector inclined 45° to the axis of symmetry — it is found
that for a given value of the rigidity modulus, the former of
these quantities is decreased and the latter increased by com­
pressibility. The solution has also been generalized so as to
apply to the case in which the potential of the disturbing
forces is any spherical harmonic of degree not less than 2.

5. In his adaptation of the "short method of determining
orbits" to the direct computation of a parabola for comets,
Professor Leuschner derives the geocentric distance \( r \) at a fixed
date from the equation

\[
(z - p')^2 - \frac{n}{[(z - c)^2 + s^2]^1/2} - q^2 = 0,
\]

where \( z = \rho / R \), and \( R \) is the distance of the sun. \( p', n, q^2, \)
c, and \( s \) are auxiliary quantities depending on observed coordi­
nates and other data. \( c \) and \( s \) are the sine and cosine of the
angle \( \psi \) subtended at the observer by the arc between the
comet and the sun.

The solutions are given by the intersections of the parabola
\( y = z^2 \) and the curve \( y = n/\sqrt{[z - c]^2 + s^2} - q^2 \) for which \( z' \)
is real and positive, where \( z = z - p' \) and \( c' = c - p' \).

There will be either one or three solutions. Three solutions
exist, if either

\[ p' > 0, \quad c > 0; \]

or

\[ p' > 0, \quad c < 0, \quad 90^\circ < \psi < 125^\circ 16'; \]
or \[ p' < 0, \quad e > 0, \quad 0^\circ < \psi < 54^\circ 44', \]
and if

\[ \left( \frac{p}{3} \right)^3 + \left( \frac{q}{2} \right)^2 < 0, \]

where

\[ \frac{p}{3} = \frac{1}{81} \left[ 9(2s^2 + q^2) - 7c^2 \right]; \quad \frac{q}{2} = \frac{5c'}{9} \left[ \frac{p}{3} + \frac{1}{9} \left( m^2 - \frac{11}{10} q^2 \right) \right] \]

and

\[ m^2 = c^2 + s^2. \]

For solution the equation in \( z \) is written in the form

\[ y' = (\zeta + \epsilon)^2 - (\eta - q^2) = f(\vartheta) = 0, \]

where

\[ \zeta = s \tan \vartheta; \quad \eta = \frac{n}{s} \cos \vartheta. \]

A convenient graphical solution is proposed for the solution of \( f(\vartheta) = 0 \). Then

\[ \rho / R = z = s \tan \vartheta + c. \]

Geocentric distances correct to four or five decimals result from the graphical solution. Further decimals may be obtained by a simple differential correction.

In practice no case with three solutions has been encountered.

C. A. Noble,
Secretary of the Section.

THE WINNIPEG MEETING OF THE BRITISH ASSOCIATION.

The seventy-ninth annual meeting of the British Association for the advancement of science was held in Winnipeg August 25 to September 1. Fourteen hundred members and associates were in attendance. The opening event was the address of the President of the Association, Sir J. J. Thomson, on Wednesday evening, August 25, in which he gave an account of some of the more recent developments in physics and in his opening remarks took occasion to urge a closer union between mathematics and physics and to emphasize the advantages of