THE OCTOBER MEETING OF THE AMERICAN MATHEMATICAL SOCIETY.

THE one hundred and forty-fifth regular meeting of the Society was held in New York City on Saturday, October 30, 1909, extending through a morning and an afternoon session. About forty persons were in attendance, including the following twenty-seven members of the Society:

Professor G. D. Birkhoff, Professor E. W. Brown, Professor A. S. Chessin, Professor F. N. Cole, Miss L. D. Cummings, Professor L. P. Eisenhart, Professor T. S. Fiske, Professor C. C. Grove, Professor C. N. Haskins, Professor W. H. Jackson, Mr. S. A. Joffe, Dr. L. C. Karpinski, Professor Edward Kasner, Professor C. J. Keyser, Mr. W. C. Krathwohl, Professor W. W. Landis, Professor J. H. Maclagan-Wedderburn, Mr. H. H. Mitchell, Professor G. D. Olds, Professor W. F. Osgood, Mr. H. W. Reddick, Dr. W. M. Strong, Professor Oswald Veblen, Mr. C. B. Walsh, Mr. H. E. Webb, Professor H. S. White, Miss E. C. Williams.

Vice-President Edward Kasner occupied the chair at the morning session, being relieved at the afternoon session by Ex-Presidents Osgood and White. The Council announced the election of the following persons to membership in the Society: Dr. H. T. Burgess, University of Wisconsin; Professor H. H. Dalaker, University of Minnesota; Mr. G. C. Evans, Harvard University; Mr. Louis Gottschall, New York City; Dr. J. V. McKelvey, Cornell University; Miss H. H. MacGregor, Yankton College; Mr. H. H. Mitchell, Princeton University; Mr. U. G. Mitchell, Princeton University; Mr. R. R. Shumway, University of Minnesota; Dr. H. L. Slobin, University of Minnesota; Mr. I. W. Smith, University of North Dakota. Four applications for admission to membership in the Society were received.

The list of nominations for officers and other members of the Council to be placed on the official ballot for the annual meeting was reported. Mr. C. B. Upton was appointed Assistant Librarian of the Society.

In memory of Ex-President Simon Newcomb the following resolutions, presented by the Council, were adopted by the Society:

Resolved: That the American Mathematical Society record its great regret at the loss to the Society and to Science occasioned by the death of Professor Simon Newcomb.

As a member of the Society from the time when its scope became national in character and as one of its early Presidents, he showed his interest in its development. His advice and influence greatly contributed to the success which has attended its efforts toward the organization and progress of mathematical science throughout the American continent.

His achievements in many branches of thought and particularly in astronomical science will entitle him to high rank amongst those who have labored to advance knowledge and to place it within the reach of every student.

Resolved: That these resolutions be recorded in the minutes of the Society and that a copy of them be sent by the Secretary to the family of the late Professor Newcomb.

The following papers were read at this meeting:

(1) Professor C. N. HASKINS: "Note on the extremes of functions."

(2) Professor P. A. LAMBERT: "On the solution of linear differential equations."

(3) Professor FLORIAN CAJORI: "A note on the history of the slide rule."

(4) Professor CARL RUNGE: "A hydrodynamic problem treated graphically."

(5) Professor EDWARD KASNER: "The motion of particles starting from rest."

(6) Professor G. A. MILLER: "Note on the groups generated by two operators whose squares are invariant."

(7) Professor C. N. MOORE: "On the uniform convergence of the developments in Bessel functions of order zero."

Professor Runge was introduced by the Secretary. In the absence of the authors, the papers of Professor Lambert, Professor Cajori, Professor Miller, and Professor Moore were read by title.

Professor Miller's paper appears in full in the present number of the BULLETIN. Abstracts of the other papers follow below. The abstracts are numbered to correspond to the titles in the list above.

1. Professor Haskins discusses a representation of the maximum maximorum and minimum minimorum of a function of a real variable as the limit of a definite integral. If in the interval $0 \leqq x \leqq 1, f(x)$ is continuous and m and M respectively its greatest lower and least upper bounds, and

$$J(z) = \tfrac{1}{2} \log \int_0^1 e^{zf(x)}\, dx,$$

then

$$\lim_{z=+\infty} J(z) = M, \quad \lim_{z=-\infty} J(z) = m,$$

and

$$\lim_{z=0} J(z) = \int_0^1 f(x) dx.$$

Moreover $J(z)$ is a monotonic increasing function of z, a fact which is proved by repeated use of the integral inequality of Schwarz. The theorems are closely related to those recently published by Dunkel,* but the proofs proceed by methods of the integral calculus.

2. The object of Professor Lambert's paper is to present a method of building up the solution of ordinary differential equations with algebraic coefficients. The method consists of the following steps: (a) Break up the given differential equation

$$(1) \qquad f\left(x, y, \frac{dy}{dx}, \frac{d^2y}{dx^2}, \ldots, \frac{d^ny}{dx^n}\right) = 0$$

into two parts, one of which equated to zero gives a differential equation which is readily solved by the usual methods, and introduce a parameter t as a factor of the other part, so that the given equation becomes

$$(2) \quad f_1\left(x, y, \frac{dy}{dx}, \cdots, \frac{d^ny}{dx^n}\right) + f_2\left(x, y, \frac{dy}{dx}, \cdots, \frac{d^ny}{dx^n}\right) t = 0.$$

(b) Assume that

$$(3) \qquad y = A + Bt + Ct^2 + Dt^3 + \cdots + Nt^n + \cdots,$$

where $A, B, C, D, \cdots, N, \cdots$ are undetermined functions of x, makes equation (2) an identity. Determine $A, B, C, D, \cdots, N, \cdots$ by solving the differential equations found by equating

* *Annals of Mathematics*, sec. 2, vol. 11 (1909), p. 26.

to zero the coefficients of the successive powers of t in this identity.

(c) Substitute these values of $A, B, C, D, \cdots, N, \cdots$ in (3) and place t equal to unity; then

$$(4) \qquad y = A + B + C + D + \cdots + N + \cdots,$$

provided this series terminates or is convergent, is a solution of the differential equation.

3. It is shown by Professor Cajori that the earliest German description of the rectilinear slide rule, found in Leupold's Theatrum arithmetico-geometricum, Leipzig, 1727, and copied by Leupold from an old manuscript of hitherto unknown authorship, is a translation of passages in Seth Partridge's Description and use of an instrument called the double scale of proportion, which appeared at London in several editions. The earliest edition bears the date 1662, and not 1671, as hitherto supposed.

4. The amount of water flowing from a reservoir of infinite depth over a weir of given section may be approximately found by graphical methods, while there do not exist any methods of treating the problem analytically. In Professor Runge's paper the viscosity is neglected and the problem is considered in two dimensions only. The velocity potential is deduced by starting with a plausible assumption of the free surface. On this curve the velocity of the water, i. e., the gradient of the velocity potential, is known. From these data the stream lines and lines of constant potential may be constructed and any stream line may serve as a possible section of a weir. For practical purposes it is only necessary to approximate the given weir at the top, because the rest of it will have little influence on the amount of flow.

5. Professor Kasner showed that when a particle, starting from rest, is acted on by any positional field of force, it describes a path whose curvature is one third the curvature of the line of force through the given point. The case where the curvature is zero is studied separately, a simple result being obtained concerning the lowest non-vanishing derivatives. The discussion is extended to constrained motion and to systems of particles.

7. In Professor Moore's paper the following theorem is established: If in the interval $0 \leqq x \leqq 1$ the function $f(x)$ is continuous, save at a finite number of points at which it has a finite jump, and if in each interval of continuity it has a second derivative that is finite and integrable, then the development of $f(x)$ in Bessel functions of order zero will converge uniformly to the value $f(x)$ throughout the interval $0 \leqq x \leqq x_0 < c$, where c is the smallest value of x for which $f(x)$ is discontinuous.

It is a well known fact that, under much wider conditions than those imposed here, the development will converge uniformly to $f(x)$ throughout any closed interval that does not include nor reach up to a point of discontinuity of $f(x)$, and does not reach up to the origin. The uniform convergence of the series in the neighborhood of the origin, however, does not seem to have been discussed.

F. N. COLE,
Secretary.

NOTE ON THE GROUPS GENERATED BY TWO OPERATORS WHOSE SQUARES ARE INVARIANT.

BY PROFESSOR G. A. MILLER.

(Read before the American Mathematical Society, October 30, 1909.)

LET s_1, s_2 be any two operators which satisfy the conditions

$$s_1^{-1}s_2^2s_1 = s_2^2, \quad s_2^{-1}s_1^2s_2 = s_1^2.$$

From these equations it results directly that both s_1^2 and s_2^2 are invariant under the non-abelian group G generated by s_1, s_2. On the other hand the equations

$$s_1^{-1} \cdot s_1s_2^{-1} \cdot s_1 = s_2^{-1}s_1 = (s_1s_2^{-1})^{-1} \cdot s_1^2s_2^{-2} = s_2^{-1} \cdot s_1s_2^{-1} \cdot s_2,$$

$$s_2^{-1} \cdot s_2s_1^{-1} \cdot s_2 = s_1^{-1}s_2 = (s_2s_1^{-1})^{-1} \cdot s_1^{-2}s_2^2 = s_1^{-1} \cdot s_2s_1^{-1} \cdot s_1$$

show that the abelian group generated by the three operators $s_1^2, s_2^2, s_1s_2^{-1}$ is invariant under G, and also that the quotient group of G with respect to the group H generated by s_1^2, s_2^2 is dihedral. Hence it results also that H is the central of G whenever $s_1s_2^{-1}$ is of odd order. When $s_1s_2^{-1}$ is of even order, the central of G is either H or it includes H as a subgroup of half its order. Since the quotient group of G with respect to an abelian invariant subgroup is dihedral G must be solvable.