7. In Professor Moore's paper the following theorem is established: If in the interval $0 \leqq x \leqq 1$ the function $f(x)$ is continuous, save at a finite number of points at which it has a finite jump, and if in each interval of continuity it has a second derivative that is finite and integrable, then the development of $f(x)$ in Bessel functions of order zero will converge uniformly to the value $f(x)$ throughout the interval $0 \leqq x \leqq x_0 < c$, where $c$ is the smallest value of $x$ for which $f(x)$ is discontinuous.

It is a well known fact that, under much wider conditions than those imposed here, the development will converge uniformly to $f(x)$ throughout any closed interval that does not include nor reach up to a point of discontinuity of $f(x)$, and does not reach up to the origin. The uniform convergence of the series in the neighborhood of the origin, however, does not seem to have been discussed.

F. N. Cole,
*Secretary.*

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# NOTE ON THE GROUPS GENERATED BY TWO OPERATORS WHOSE SQUARES ARE INVARIANT.

BY PROFESSOR G. A. MILLER.

(Read before the American Mathematical Society, October 30, 1909.)

Let $s_1, s_2$ be any two operators which satisfy the conditions

$$s_1^{-1}s_2^2s_1 = s_2^2, \quad s_2^{-1}s_1^2s_2 = s_1^2.$$

From these equations it results directly that both $s_1^2$ and $s_2^2$ are invariant under the non-abelian group $G$ generated by $s_1, s_2$. On the other hand the equations

$$s_1^{-1} \cdot s_1s_2^{-1} \cdot s_1 = s_2^{-1}s_1 = (s_1s_2^{-1})^{-1} \cdot s_1^2s_2^{-2} = s_2^{-1} \cdot s_1s_2^{-1} \cdot s_2,$$

$$s_2^{-1} \cdot s_2s_1^{-1} \cdot s_2 = s_1^{-1}s_2 = (s_2s_1^{-1})^{-1} \cdot s_1^{-2}s_2^2 = s_1^{-1} \cdot s_2s_1^{-1} \cdot s_1$$

show that the abelian group generated by the three operators $s_1^2, s_2^2, s_1s_2^{-1}$ is invariant under $G$, and also that the quotient group of $G$ with respect to the group $H$ generated by $s_1^2, s_2^2$ is dihedral. Hence it results also that $H$ is the central of $G$ whenever $s_1s_2^{-1}$ is of odd order. When $s_1s_2^{-1}$ is of even order, the central of $G$ is either $H$ or it includes $H$ as a subgroup of half its order. Since the quotient group of $G$ with respect to an abelian invariant subgroup is dihedral $G$ must be solvable.

The object of the present note is to show clearly that $G$ may be regarded as a generalization of the dihedral group which includes the earlier generalization obtained by considering the groups generated by two operators which have a common square.* If two operators have a common square, this square is clearly invariant under the group generated by these operators; but if the squares are invariant under this group they evidently are not necessarily the same. From this it follows directly that the present generalization includes the earlier one, and it gives rise to an almost equally elementary category of groups as a result of the equations established in the preceding paragraph. If two operators have a common square, it is known that the product of either one into the inverse of the other is transformed into its inverse by each of the operators. The analogous theorem as regards the operators under consideration may be expressed as follows:

*When each of two operators is commutative with the square of the other, the product of one into the inverse of the other is transformed by each of the two operators into its inverse multiplied by an invariant operator under the group generated by the two operators.*

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# THE SOLUTION OF THE EQUATION IN TWO REAL VARIABLES AT A POINT WHERE BOTH THE PARTIAL DERIVATIVES VANISH.

BY DR. L. S. DEDERICK.

(Read before the American Mathematical Society, September 14, 1909.)

IF $F(x, y)$ is a real function of the real variables $x$ and $y$ which is continuous at and near the point $(x_0, y_0)$, and vanishes at this point, but has one first order partial derivative at the point not equal to zero, there are a number of well-known theorems about the existence of other values of $x$ and $y$ satisfying the equation

$$(1) \qquad F(x, y) = 0,$$

* *Archiv der Mathematik und Physik*, vol. 9 (1905), p. 6.