line (the system ${}^1R_1$) and for the plane (the system ${}^2R_2$). In particular, he showed how his planar descriptive system could be logically combined with linear projective axioms in such a way that the resulting system might be descriptive or projective, but not necessarily either.

25. In his second paper, Mr. Schweitzer presented a preliminary report on a study of the memoirs of F. Riesz * and H. Hahn † in relation to the generalization of Grassmann's extensive algebra ‡ to a denumerably infinite number of dimensions.

H. E. SLAUGHT,
Secretary of the Section.

THE SIXTY-FIRST MEETING OF THE AMERICAN ASSOCIATION FOR THE ADVANCEMENT OF SCIENCE.

THE sixty-first meeting of the American Association for the Advancement of Science was held in Boston during the convocation week, December 27, 1909, to January 1, 1910. The president of the meeting was President D. S. Jordan, of Stanford University. The address of the retiring president, Professor T. C. Chamberlin, entitled "A geologic forecast of the future opportunities of our race" was given in Sanders Theatre, Cambridge, on the evening of the opening day.

Comparatively few papers on pure mathematics appeared on the separate program of Section A (mathematics and astronomy) because of the fact that the American. Mathematical Society held its annual meeting in affiliation with the Association. The address of the retiring vice-president, Professor C. J. Keyser, of Columbia University, entitled "The thesis of modern logistic," was given on Wednesday morning at a joint session of Section A and the American Mathematical Society. At the same session Professor D. E. Smith presented a report on the work of the International Commission on the teaching of mathematics.

Another joint session was held on Tuesday afternoon under the auspices of the mathematicians and the physicists. During

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* *Math. Naturw. Berichte aus Ungarn*, 1905, pp. 309, 341-343, etc.
† *Wiener Berichte*, Abt. II^A, vol. 116, pp. 601, 609-610, 642, etc.
‡ See also *American Journal*, October, 1909, pp. 365-410.

this session Professor C. Runge, Kaiser Wilhelm exchange professor of mathematics at Columbia University, read a paper "On the determination of latitude and longitude in a balloon," and Professor E. W. Brown read the first of his two papers in the list below. These joint sessions constituted the most noteworthy features of the program of Section A, and it is to be hoped that they may have tended to attract more investigators to the border land between mathematics and physics, where our country seems to be especially in need of workers.

The officers of Section A were: vice-president, E. W. Brown; secretary, G. A. Miller; councilor, G. B. Halsted; member of the general committee, H. W. Tyler; sectional committee, C. J. Keyser, E. W. Brown, G. A. Miller, E. O. Lovett, Harris Hancock, F. R. Moulton, Winslow Upton. On the recommendation of the sectional committee the following twenty-two members of the American Mathematical Society were elected fellows of the Association: R. P. Baker, W. E. Brooke, Thomas Buck, Arthur Crathorne, I. M. DeLong, C. E. Dimick, F. J. Dohmen, J. F. Downey, L. P. Eisenhart, J. C. Fields, B. F. Finkel, F. L. Griffin, A. G. Hall, C. N. Haskins, T. F. Holgate, J. I. Hutchinson, D. N. Lehmer, W. D. MacMillan, C. A. Noble, F. G. Reynolds, Charlotte A. Scott, A. W. Smith.

The following twenty papers were read before the Section:

(1) Professor C. J. KEYSER: "The thesis of modern logistic."

(2) Professor C. RUNGE: "On the determination of latitude and longitude in a balloon."

(3) Professor E. W. BROWN: "On certain physical hypotheses sufficient to explain an anomaly in the moon's motion."

(4) Professor D. E. SMITH: "The work of the International Commission on the teaching of mathematics."

(5) Mr. C. G. ABBOT: "The value of the solar constant of radiation."

(6) Professor F. W. VERY: "A new mode of measuring the intensities of spectral lines."

(7) Professor F. W. VERY: "The absorption of light by the ether of space."

(8) Professor F. W. VERY: "The fireball of October 7, 1909."

(9) Professor E. W. BROWN: "On a recent hypothesis and the motion of the perihelion of Mercury."

(10) Professor J. A. MILLER and Mr. W. R. MARRIOTT: "The heliocentric position of certain coronal streams."

(11) Professor D. P. TODD: "The mutual relation of magnifying power, illumination, aperture, and definition in telescopic work."

(12) Professor G. B. HALSTED: "La contribution non-euclidienne à la philosophie."

(13) Mr. H. E. WETHERILL: "Declination of the moon for Greenwich mean time."

(14) Mr. H. W. CLOUGH: "Meteorological waves of short period and allied solar phenomena."

(15) Professor MILTON UPDEGRAFF: "Recent work with the 6-inch transit circle of the United States Naval Observatory."

(16) Mr. V. M. SLIPHER: "Peculiar star spectra indicating selective absorption of light in space."

(17) Mr. R. M. STEWART: "Personality with the transit micrometer."

(18) Professor F. W. VERY: "Water vapor on Mars."

(19) Professor C. L. DOOLITTLE: "The existence of anomalous fluctuations in the latitude as shown by simultaneous observations with the zenith telescope and the reflex zenith tube of the Flower Observatory."

(20) Mr. LEON CAMPBELL: "Visual observations of variable stars at the Harvard College Observatory."

In the absence of their respective authors the papers by Professor Todd, Mr. Wetherill, and Mr. Slipher were read by title, and that of Mr. Stewart was read by Dr. O. J. Klotz. The remaining papers were read by their authors. Professor Keyser's vice-presidential address appeared in *Science*, December 31, 1909. Abstracts of the other papers of mathematical interest are given below. They bear numbers corresponding to those of the titles in the list given above.

2. Professor Runge remarked that the problem of finding the geographical position in a balloon from observations of the sun is very different from the same problem on a ship for the reason that in a balloon there is no dead reckoning. In a balloon the only way of getting the geographical position from the sun is by observing both altitude and azimuth at the same time. The accuracy with which the azimuth of the sun may be observed is rather rough; it would be difficult to obtain it within less than one tenth of a degree. Therefore the reduction of the observations need not be very accurate. At the

same time it is essential that the reduction should be made very quickly. For the time since the moment the observations were taken introduces an uncertainty which may be expressed by the area of a circle whose radius is equal to the distance through which the balloon may have traveled.

One naturally would therefore turn to graphical methods for the reduction of the observations. The reduction consists in finding the latitude ϕ and the hour angle t from the declination δ, the azimuth a, and the altitude h. Professor Runge proposes to find first the latitude ϕ from δ, a, h and then the hour angle t from δ, a, ϕ. In both cases we have to deal with the representation of an equation between four variables. Both of these equations may be written in the following form:

$$f(p) + h(r, s)g(q) = k(r, s),$$

where p, q, r, s denote the four variables. That is to say, two of the variables enter the equations in separate functions $f(p)$, $g(q)$, and the equation is linear in these functions, the coefficients being any functions of the other two variables. Equations of this kind may be represented graphically by the "méthode des points alignés" of Maurice d'Ocagne,* taking $f(p)$ and $g(q)$ as line coordinates, making $f(p)$ equal to the ordinate of the point of intersection of the straight line with the axis of ordinates and $g(q)$ equal to the gradient of the straight line, that is, the tangent of its angle with the axis of abscissas. In this way the rectangular coordinates of the point whose equation in line coordinates is the given equation become

$$x = h(r, s), \quad y = k(r, s).$$

For any given value of p the different values of q correspond to straight lines which form a pencil of rays whose center is on the axis of ordinates at the particular value defined by p, and any alteration of p would simply shift the center along the axis without altering the pencil of rays in any other way. The whole diagram may therefore be obtained by drawing two figures, one containing the curves $r =$ constant and $s =$ constant and the other containing the pencil of rays, and placing these two figures in the proper way one over the other. It so happens that the variable p is the declination of the sun which

* Traité de Nomographie.

may be regarded as constant during the ascent of the balloon. The aeronaut would therefore merely use a definite superposition of figures. These are photographed on transparent plates and a blue print is taken by copying the plates, one after another, on the same paper in proper position.

The aeronaut has one blue print to read off the latitude, a second to read off the hour angle after he has found the latitude. The equations are

$$\sin \delta + \cos \phi \cos h \cos a = \sin \phi \sin h,$$

$$\tan \delta + \sec \phi \sin t \cot a = \tan \phi \cos t.$$

In the first equation the curves $\phi =$ const. and $h =$ const. are the ellipses $x = \cos \phi \cos h$, $y = \sin \phi \sin h$. In the second equation the curves $\phi =$ const. and $t =$ const. are the confocal ellipses and hyperbolas

$$x = \sec \phi \sin t, \quad y = \tan \phi \cos t.$$

3. Newcomb has shown that there is a difference between the observed and the theoretical position of the moon which can be roughly represented by a term of period about 270 years and coefficient 13″. In the present paper Professor Brown examined numerous hypotheses sufficient to explain the term, in order to clear the ground of those which seemed to be of doubtful value and to bring forward those which appeared sufficiently reasonable to merit tests from observations of a different nature. He gave an account of three of these hypotheses, stating that a minute libration of the moon would be sufficient provided it took place in the moon's equator and had the proper period. The supposition of magnetic attraction practically demanded a periodic change in the magnetic moment of the earth or of the moon. If this were rejected, it was necessary to suppose that the mean place of the lunar magnetic axis was near the lunar equator and that the oscillations of its position took place in the plane of the equator. The observed secular change of the earth's magnetic axis could not produce the phenomenon without demanding a larger motion of the lunar perigee than observations warrant. On the border line between two sets of hypotheses is a curious fact, namely, that if the period of the solar rotation coincides very nearly with one of the principal lunar periods, a minute equatorial ellipticity of the sun's mass would be sufficient to explain the term. So

far as known these hypotheses do not conflict with any observed phenomena, but they cause some theoretical difficulties.

4. The International Commission on the teaching of mathematics was suggested some years ago but the first steps in its organization were not taken until April, 1908. At that time the Fourth International Congress of Mathematicians, then in session in Rome, empowered Professor Klein of Göttingen, Sir George Greenhill of London, and Professor Fehr of Geneva to appoint such a commission, and to arrange for it to report at the next congress to be held at Cambridge in 1912. As a result three commissioners have been selected from each of the leading countries and the work has actively begun. Those of the United States are Professors D. E. Smith, W. F. Osgood, and J. W. A. Young. It is expected that each of these countries will submit a very full report of the nature of the work in mathematics, from the kindergarten through the college, with some discussion of the range of advanced work in the universities. In the United States the investigation is carried on by means of fifteen committees, each divided into subcommittees. Professor Smith stated that about two hundred and seventy-five people are engaged in the work and the subcommittee reports will be submitted during the present winter. The committee reports will be submitted before the summer of 1910, and the national report by Easter, 1911. This paper is to appear in the *American Mathematical Monthly.*

9. Professor Brown's second communication consisted of a brief account of the hypotheses of Seeliger brought forward to account for the outstanding large residual in the motion of the perihelion of Mercury and the small residuals in the secular motions of the four minor planets. An analysis of the nature of the three hypotheses and a comparison of the number of arbitrary constants introduced, with the number of residuals to be accounted for, were also given.

10. Professor Halsted's memoir describes the meaning and growth of non-euclidean geometry, sketches its history and founders, and points out that philosophy has found a new criterion in this subject. The memoir is to appear in French in the *Mémoires de la Société des Sciences physiques et naturelles de Bordeaux.*

The next regular meeting of the Association will be held at the University of Minnesota under the presidency of Professor A. A. Michelson, of the University of Chicago. Professor E. H. Moore, of the University of Chicago, was elected vice-president and chairman of Section A; Professor G. A. Miller, of the University of Illinois, continues as secretary. Professor E. R. Smith, of the Brooklyn Polytechnic Preparatory School, was elected member of the sectional committee for five years.

G. A. MILLER,
Secretary of Section A.

SHORTER NOTICES.

Rara Arithmetica. By DAVID EUGENE SMITH, Teachers College, Columbia University. Second edition. Boston, Ginn and Company, 1909. xviii + 507 pp. Cloth. Price, $4.50.

THE title page of this splendid volume modestly states that "the work is a catalogue of the arithmetics written before the year 1601 with a description of those in the library of George Arthur Plimpton of New York." Another appropriate title might be, A brief history, on the bibliographical plan, of the genesis and content of sixteenth century arithmetic.

As a bibliography this work is more extensive than any of its predecessors, and is nearly complete for the formative period between 1472 and 1601. There are mentioned over five hundred and fifty different works, which number swells nearly to twelve hundred by the inclusion of the various editions. About four hundred and fifty of the different books are genuine arithmetics, while the others deal partially with algebra, astrology, or the calendar. In determining the significance of this number one thinks at once of De Morgan's Arithmetical Books, the best of the earlier authorities on the subject, and recalls that this work gives only seventy arithmetics printed before 1600. The list in Professor Smith's Rara Arithmetica is much more extensive than those of Graesse, Hain, and Coppinger, and contains more Italian titles than are given by Riccardi in his Bibliotheca Mathematica Italiana and more German ones than are included by Murhard in his Bibliotheca Mathematica.

But this work is more than a scholarly, well edited digest of all the earlier bibliographies; it is the result of the examination