tral and principal planes and the classification of conicoids are then briefly treated. There is a short discussion of the invariants of the general equation under a transformation from one orthogonal system of axes to another orthogonal system.

The text proper is followed by six short tables, which deal with algebraic and trigonometric formulas, derivatives and partial derivatives, four-place table of logarithms of a few numbers, lengths of arcs in radians, and the letters of the Greek alphabet. The book is concluded by a set of nine very good plates showing the silk thread figures of the ruled surfaces of the second order, as well as the usual plaster models of the conicoids. Indeed, one of the best features of the book is to be found in the excellence of the numerous figures. The young man in whose hands this text is placed will probably note first of all that it is small enough to fit into his pocket. By employing rather thin backs and paper that is not too heavy and by lessening the margins, the size and weight of the volume have been reduced to a minimum. Perhaps the strongest feature of the book is to be found in the abundant supply of examples. After each bit of theory there are some exercises, and at the end of each of the longer chapters there is a set of about fifty carefully graded problems.

E. B. COWLEY.

Leçons sur les Fonctions définies par les Equations différentielles du premier Ordre. Par PIERRE BOUTROUX. Paris, Gauthier-Villars, 1908. 190 pp.

THE little volume bearing the above title is one of the series of monographs on the theory of functions published under the editorship of E. Borel. The author's aim is to set forth the theory of functions defined by a differential equation as based on the work of Painlevé. He abandons the "local point of view" of Cauchy and studies the ensemble and form of the integral not only in the neighborhood of a point but in general. The particular question discussed is one raised by Painlevé, viz., how does the solution behave when the initial point x_0 at which it is considered varies from point to point in an arbitrary manner.

The book is divided into five chapters. Chapter I presents the fundamental notions. After a review of the usual theory of singular points the following theorem of Painlevé's is demonstrated: As x varies from x_0 to $\bar{x}$ along a curve L joining x_0 to $\bar{x}$ an integral Y of the equation

$$(1) \qquad \frac{dy}{dx} = \frac{P(x, y)}{Q(x, y)}$$

(P and Q being polynomials in y) which takes in x_0 the value C will approach a definite value.

From this theorem the following one is deduced at once: Aside from fixed singular points, an integral of (1) can have only poles or algebraic critical points for singularities.

Another important theorem also due to Painlevé is given in this chapter: If P and Q are polynomials in y and algebraic in x and if the integrals are multiform functions of n branches then equation (1) can be reduced to Riccati's equation by a rational change of variable. One can always determine by a finite number of operations if this change of variable is possible. A discussion of multiform functions of an infinite number of branches and the corresponding Riemann surfaces closes the chapter.

Chapter II deals with the growth and behavior of a branch of an integral. The subject here discussed is the manner in which an integral increases as x increases. The problem is to determine whether an integral increases more slowly or more rapidly than a power of $|x|$ or than an exponential function. The subject is treated in quite a fascinating manner, but the presentation must be read in order to be appreciated.

Chapter III is on classification of singularities. Transcendental singular points are divided into two classes, those directly critical and those indirectly critical. A point x is directly critical if when an elementary circuit is described about x an infinity of branches are permuted. It is indirectly critical if an infinity of branches are permuted when an elementary circuit is described not about x properly speaking but about an infinity of critical points which are converging toward x as a limit. Following the above definitions the singular points of the equation

$$(2) \qquad 2z\frac{dz}{dx} = \alpha x + \beta z$$

are discussed in considerable detail under various assumptions, and the manner in which the branches permute themselves is shown.

After a discussion of other particular examples a few pages are devoted to a more minute classification of transcendental singularities.

Chapter IV considers the singular points of Briot and Bouquet. The equation which has singularities of this type is connected with the equation above by introducing a parameter μ which put equal to zero gives equation (2) and put equal to unity gives the equation under consideration. By allowing μ to vary, the intimate relation between the singularities of equation (2) and those of Briot and Bouquet is established.

Chapter V discusses some of the relations which exist between the singularities of the same equation.

The volume closes with a note of fifty pages by Painlevé: "On the differential equations of the first order whose general integral has only a finite number of branches."

C. L. E. MOORE.

Sur les premiers Principes des Sciences mathématiques. Par P. WORMS DE ROMILLY. Paris, A. Hermann, 1908. 8vo. 51 pp. 2.50 fr.

THIS essay undertakes to give an account of the recent work on the foundations of mathematics. The author concludes that the only branch of mathematics completely applicable to natural phenomena is arithmetic, since it depends solely upon the numeration of objects, and makes no hypothesis regarding their nature. Geometry, on the other hand, imposes upon them certain purely ideal hypotheses which indeed may differ so as to produce at least three systems of geometry, the system which nature is built upon being possibly that of Euclid, possibly otherwise. The contrast drawn here between the external validity of arithmetic as over against that of geometry is a little difficult to reconcile with the explanations devoted by the author to the varying systems of axioms on which arithmetic may be based. In fact he distinctly speaks of diverse systems of numeration. We might inquire, for example, are objects subject to the archimedean axiom or not?

A disproportionate amount of space is devoted to the setting forth of some seven foundations upon which geometry may be based, and not quite so much to mechanics. The reason for this is the underlying thesis which the author seeks to prove. He examines the different modes of grounding geometry and concludes they are all à priori and inapplicable to real objects