Synthetische Theorie der Cliffordschen Parallelen und der linearen Linienörter des elliptischen Raumes. Von WOLFGANG VOGT. Leipzig und Berlin, Teubner, 1909. vii + 58 pp.

THE parallels of the hyperbolic space seemed for many years to furnish the only possible extension of the notion of parallelism derived from euclidean geometry. But about forty years ago the brillant Clifford discovered in elliptic space straight lines which possess most of the properties of euclidean parallels, but differ from them in being skew. Two lines are right (or left) parallel if they cut the same right (or left) generators of the absolute. After Clifford's death, F. Klein and R. S. Ball made extensive contributions to the knowledge of the properties of these lines. More recently E. Study and J. L. Coolidge have been studying the subject. Analytic, synthetic, and differential geometry and vector analysis have been employed by various investigators. But the author of the present paper thinks that no *purely* synthetic treatment has appeared, although such a method is not only possible, but very suitable for the problem. He therefore aims to give the theory of the Clifford parallels in a purely synthetic form.

The article begins with an explanation of the notion of the "winding" of two lines. The Clifford parallels are defined as lines which have more than two common perpendiculars. The nets of parallels of both "windings" are deduced from the v. Staudt-Lüroth theory of line "nets" with imaginary directrices. At the same time, their projective connections with the absolute polar space and with each other are obtained. By elementary methods, it is shown that theorems on euclidean parallels hold true for the Clifford parallels. Other propositions due to modern writers are also demonstrated. The first part of the article is concluded by a brief discussion of the kinematics of elliptic space.

In the second chapter the three sections deal with ruled systems of the second order, the linear complex, and the linear congruence. The projective properties of these linear line loci are assumed and their metrical relations are deduced. Special attention is paid to the appearance of parallels. The parallel cone or the asymptotic cone of a ruled surface of the second order splits, in the elliptic space, into two cones coaxial with the surface. But two arbitrary coaxial cones do not form the parallel cones of a ruled surface of the second order unless certain conditions are fulfilled. A classification of ruled surfaces of the second order is made according to the form taken by their cones. It is interesting to compare this with the classifications of quadrics in elliptic space given by J. L. Coolidge (Non-Euclidean Geometry, page 156) and T. J. I'a. Bromwich ("The classification of quadric loci," *Transactions*, volume 6, 1905). In these articles the principles of classification are entirely different from that employed here.

In the section dealing with linear complexes, right and left complexes are distinguished, the existence of "diameter parallel nets" is proved, and the appearance of parallels in the linear complex and in the corresponding null space are investigated. Of special interest is the parallel complex, which possesses a whole net of axes and admits ∞^4 motions carrying it into itself, while the ordinary complex has only ∞^2. The article is concluded by a discussion of the properties of the general linear congruence and some of its special forms.

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Das Gruppenschema für züfallige Ereignisse. Von HEINRICH BRUNS. Des XXIX Bandes der *Abhandlungen der Mathematisch-Physikalischen Klasse der Königl. Sächischen Gesellschaft der Wissenschaften,* No. VIII. Leipzig, B. G. Teubner, 1906. Pp. 579–628.

THIS monograph is an extension of the brief development of the subject in the eighteenth lecture contained in the treatise by Bruns on Wahrscheinlichkeitsrechnung und Kollektivmasslehre. It is assumed in setting the simplest problem of the work that n balls are drawn, one at a time, from a bag containing balls of various colors, and that each time the ball drawn is returned to the bag before another drawing is made. The n consecutive drawings are called a draw series (Zugreihe) indicated by $Z(n)$. The draw series is written in the form

$$(1) \qquad Z(n) = z_1 z_2 \cdots z_n,$$

where z_h denotes the hth drawing.

If the draw series are collected into sets of s with subscripts 1 to s, 2 to $s+1$, 3 to $s+2$, etc., the sets of s are called s-membered draw groups and the symbol $G(s)$ is used to designate such a group. In the formation of such groups, the author distinguishes between what he calls linear and cyclical groups. If, from (1), we take merely z_1 to z_s, z_2 to z_{s+1}, $\cdots$, z_{n-s+1} to z_n,