THE APRIL MEETING OF THE SAN FRANCISCO SECTION.

THE nineteenth regular meeting of the San Francisco Section of the American Mathematical Society was held at Stanford University on Saturday, April 8, 1911. The following members were present:

Professor R. E. Allardice, Professor H. F. Blichfeldt, Professor R. L. Green, Professor Charles Haseman, Professor M. W. Haskell, Professor L. M. Hoskins, Mr. C. Kuschke, Professor D. N. Lehmer, Professor J. H. McDonald, Mr. G. F. McEwen, Professor H. C. Moreno, Professor R. E. Moritz, Professor C. A. Noble, Professor E. W. Ponzer, Professor S. D. Townley.

Professor Lehmer occupied the chair. The following papers were read at this meeting:

(1) Professor R. E. ALLARDICE: "Note on the envelope of the directrices of a certain system of conics."

(2) Professor H. F. BLICHFELDT: "Linear homogeneous groups having irreducible invariant subgroups."

(3) Mr. A. F. CARPENTER: "Geometrical interpretations of quotientiation and its inverse."

(4) Professor W. F. DURAND: "A mechanism which solves certain differential equations."

(5) Professor M. W. HASKELL: "Note on the del Pezzo quintic."

(6) Mr. C. KUSCHKE: "The equations of the 10th degree, irreducible in a given rational domain, whose groups are transitive with two as well as five systems of imprimitivity" (preliminary communication).

(7) Professor D. N. LEHMER: "Certain theorems in line geometry."

(8) Mr. W. V. LOVITT: "Transformations of partial differential equations."

(9) Professor J. H. McDONALD: "The transformation of elliptic integrals."

(10) Professor R. E. MORITZ: "On the cubes of determinants of the second, third, and higher orders."

(11) Professor E. W. PONZER: "The calculus in technical literature."

(12) Mr. L. L. SMAIL: "Toward a function theory of a certain hypercomplex variable in two units."

Professor Durand was introduced by Professor Allardice, and Mr. Lovitt and Mr. Smail by Professor Moritz. Professor Durand's paper was read by Professor Moreno and the papers of Mr. Carpenter, Mr. Lovitt, and Mr. Smail by Professor Moritz. Abstracts of the papers are given below.

1. Professor Allardice shows how the relation connecting the six distances between four points in a plane, in the form given by Cayley, may be applied almost directly to find the envelope of the directrices of a system of similar conics through three points. The envelope is a curve of the fourth class.

2. Let G' be a transitive (irreducible) collineation group of order N' in n variables, contained invariantly in a group G of order $N = N' \cdot M$ in the same variables. The more important results stated in Professor Blichfeldt's paper are: 1) No prime number greater than $n+1$ divides M; 2) Any prime factor of M which is greater than $\frac{1}{2}(n+1)$ must be a factor of N' or must be of the form $2^a \pm 1$.

3. In Mr. Carpenter's paper the quotiential coefficient

$$Q_x y = \log_{qx} qy = \lim_{h \doteq 1} \log_h \frac{f(x\,h)}{f(x)},$$

where $y = f(x)$ is first exhibited as measuring the relative rate of change of functionally connected areas, the interpretation being made in either rectangular or polar coordinates. Two loci are obtained, either one of which, when associated with $y = f(x)$, exhibits $Q_x y$ as the ratio of two lines. By a slight modification in the cartesian system, $Q_x y$ will measure the slope of $y = f(x)$. A locus is found such that the area under it is given by the process inverse to $Q_x y$, i. e., $\Pi e^{y(qx-1)}$.

4. The fundamental principle utilized by Professor Durand is that a wheel with a sharp edge rolling on a paper surface will always tend to move in its own plane. Hence, if provision is made for adjusting the inclination of this plane (always vertical) to a line in the horizontal plane taken as the axis of X, such that $a \cdot \tan \phi = f(x)$, it will result that the wheel will, in rolling, trace out the function determined by the differential

equation $a \cdot dy/dx = f(x)$. Again, if a second plane is provided upon which rolls a second sharp edged wheel representing d^2y/dx^2 and if this second plane is given a movement parallel to y and equal in amount to the y component of the movement of the wheel representing dy/dx, then it results that the various parts of the mechanism may be so connected that a tracing point will move in fulfillment of the condition

$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + y = f(x).$$

Special cases may be handled where one or more terms may disappear, one or more may become negative, or $f(x)$ become constant.

5. In this note Professor Haskell shows that when the cusps of a quintic with five real cusps are given, the cuspidal tangents can be found by a linear construction.

6. In this preliminary communication Mr. Kuschke presents the results thus far obtained in investigating the irreducible equations of the 10th degree. He derives the necessary and sufficient conditions for all possible types of abelian equations. He discusses in detail the cases

1) $$x^{10} - c = 0$$
and

2) $x^{10} - 10c^2\rho_3^2\rho_4x^6 - 25c^3\rho_3^2\rho_4^4x^4 + (15\rho_3^6\rho_4^2 - 10\rho_3\rho_4^7)c^4x^2 - c = 0,$
where

$$\rho_3 = \sqrt[5]{-\frac{e^5}{2} - \frac{e^2}{2c}\sqrt{e^6c^2 + 4e}},$$
$$\rho_4 = \sqrt[5]{-\frac{1}{c^2} + \frac{e^5}{2} + \frac{e^2}{2c}\sqrt{e^6c^2 + 4e}}, \text{ and } e \neq 0.$$

7. Professor Lehmer defines a line as at right angles to a flat pencil of rays if it is at right angles to the ray of the pencil which it intersects. He finds the locus of lines at right angles to a given line and to a given pencil to be a ruled cubic surface. He finds also the number of lines at right angles to two given pencils and meeting a given line. He determines the locus of lines at right angles to three given pencils; also

the number of lines at right angles to four given pencils. This last problem is equivalent to finding the number of lines in space such that the common perpendiculars between them and four fixed right lines meet the four fixed lines in given points.

8. Mr. Lovitt uses the infinitesimal transformation

$$X=x+\xi(x, y, z)\delta t, \quad Y=y+\eta(x, y, z)\delta t, \quad Z=z+\zeta(x, y, z)\delta t.$$

Formulas are found giving the effect of this transformation upon the partial derivatives. Recursion formulas are found and also two expressions for each partial derivative provided it is taken at least once with respect to both x and y. This furnishes a useful check. Explicit expressions are given for the changes in the first and second partial derivatives and also numerous partial differential equations of the second order which remain invariant under given transformations. The most general transformation is found which converts a linear partial differential equation in three variables, with variable coefficients, into another equation of the same form.

9. Professor McDonald's paper shows how to construct the involutions on which depend the transformations of elliptic integrals. Clifford discovered that these are furnished by the multiplication formulas. The consideration of the polygonal line inscribed and circumscribed to two conics gives, in the case of closure, an algebraic solution to the problem of Legendre and Jacobi.

10. Professor Moritz gives special forms for the cubes of determinants of the second, third, and higher orders, and demonstrates

$$\Delta_n^3=\frac{1}{(c_1d_4\cdots n_n)(c_2d_4\cdots n_n)(c_3d_4\cdots n_n)}\begin{vmatrix} A_1^2, & A_2^2, & A_3^2 \\ A_1B_1, & A_2B_2, & A_3B_3 \\ B_1^2, & B_2^2, & B_3^2 \end{vmatrix},$$

where $\Delta_n \equiv (a_1b_2c_3\cdots n_n)$ and where the large letters are the co-factors of the small letters in this determinant.

11. In a previous paper (*Science*, October 22, 1909) Professor Ponzer gave the results of an investigation into the various principles and applications of the calculus used in under-

graduate courses and the relative frequency of their occurrence. In the present paper a similar study is made of the calculus used by the practicing engineers, the more recent volumes of leading technical journals furnishing the data used. He discusses also the emphasis placed on calculus and the attitude of the engineers of different nationalities toward it.

12. Mr. Smail considers a hypercomplex number system in two units e_1, e_2, where $e_1^2 = e_1$, $e_1e_2 = e_2e_1 = 0$, $e_2^2 = e_2$. He considers the elementary conceptions and the functionality of the simpler algebraic expressions. Differentiation is defined and its limitations pointed out; integration is defined and the integral of a hypercomplex variable expressed in terms of ordinary curvilinear integrals. He finds also the conditions that make the integral independent of the path of integration.

H. C. MORENO,
Secretary of the Section.

INVARIANT CONDITIONS THAT A p-ARY FORM MAY HAVE MULTIPLE LINEAR FACTORS.

BY PROFESSOR O. E. GLENN.

(Read before the American Mathematical Society, October 29, 1910.)

§ 1. *Introduction.*

IN the case of a ternary quadratic form the vanishing of the ordinary discriminant is the necessary and sufficient condition that the form be factorable into linear factors, distinct term for term. In the case of a cubic ternary form we can obtain in various ways a set of three independent rational, integral, and homogeneous functions of the coefficients, whose simultaneous vanishing furnishes the necessary and sufficient conditions that the form be factorable into distinct linear factors. The general theorem underlying these facts has been proved by Junker,* and is that the necessary and sufficient conditions that a p-ary form of order m be a product of m distinct linear factors consist in the simultaneous vanishing of a set of $\binom{m+p-1}{m} - m(p-1) - 1$ independent seminvariant † relations among the form's coefficients.

*Junker, "Die symmetrischen Funktionen u. s. w.," *Math. Annalen*, vol. 45.

†Junker, "Ueber die Diff.-Gleichungen der Invarianten," loc. cit., vol. 64.