quadrillée sections whose ∞^2 planes envelope a surface. The ∞^3 diagonals of the quadrangles generate a complex and the third diagonal points are on ∞^2 conics. The book closes with some paragraphs on graphical tables for the representation of functions, where the author solves the problem which when proposed to him suggested this last study, namely, to determine when the equation of a certain surface of the sixth order could be put in the form of a determinant equated to zero.

The very power of Professor Stuyvaert's methods, which cover every page with italicized theorems, makes the book monotonous to read, but at the same time gives it value as a compendium. As we have pointed out throughout this review, the author of these five studies has made a large addition to the instruments of a modern geometer and at the same time pointed out the way to other fields of investigation, perhaps the best of which is to discover how to do away with the limitations of the above methods in which to a large extent the curve has to be found to fit the formulas.

E. GORDON BILL.

SHORTER NOTICES.

Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu. Par PIERRE DUHEM. Paris, A. Hermann, 1906, 1909. 8vo. 2 volumes. x + 355, iv + 469 pp. 27 francs.

IT would be difficult to find a man better fitted for the preparation of a work of this character, either by temperament or because of scholastic attainments, than the eminent professor of theoretical physics of the Faculty of Sciences at Bordeaux. Writing of a great genius whose wide range of information covered all that was then known of physics, of a man whose interest in France showed itself in divers activities, of a scholar who was in correspondence with the learned men of his time, and of an inventor whose fertile imagination was seen in a large number of helpful devices, M. Duhem had a most inspiring subject and one which he could hardly have failed to treat *con amore.*

The author confesses at the beginning to the feeling of awe that such a topic would naturally create in any serious mind that contemplates the work of this master. In his own case, he tells us, this was followed by a period of contemplation, more free from emotion, in which the idea was prominent that such a genius was after all a human being, that he was born as other men and that he struggled as others struggle, and that the problem is rather to mark the footsteps by which he advanced than to stand in awe of his great attainments. Well does he exclaim, however, "Mais ce récit, combien il est difficile de l'obtenir exact et précis."

Of the labor expended in examining the materials for this work everyone familiar with Leonardo da Vinci is aware. He left a large number of manuscripts, and in addition to those that are fairly complete there are many brief memoranda, written in his usual manner from right to left, difficult to decipher, and so mixed up as to make their sequence hopelessly obscure. It is out of this mass of material that M. Duhem has endeavored to extract the essence, and to interpret it in the light of the works of the predecessors of Leonardo, and the work of his followers in the light of these notes. From the nature of the case, therefore, this work cannot be looked upon as a life of Leonardo, nor even as a dissertation upon his works and his discoveries. It is merely what its title asserts, a set of studies,— an inquiry into the works which he had read and which had helped him attain his high position, and into the works of those who had in turn been assisted by his own fertile mind in perfecting their labors.

The author begins by a chapter setting forth the influence of Albert of Saxony upon Leonardo da Vinci, particularly as shown in the former's De cælo et Mundo. He shows by one of Leonardo's manuscripts,* begun in September, 1508,† that this work was familiar to the young scientist, and that the latter had, indeed, a copy then in hand. This point is evidenced by his note, "Alberto decelo e mundo — da fra bèrnardino," the copy either belonging to or having been made by some Brother Bernardinus. The interesting thing, however, is the manifest influence of Albertus upon Leonardo, as is shown in numerous quotations from each with respect to the spots on the moon, center of gravity, and the sphericity of the earth. M. Duhem shows that Leonardo was indebted to his predecessor for numerous initiatives, but that in nearly every case he developed an independent theory of his own. Thus, although he was led by Albertus to study the center of grav-

* Lettered F in the Bibliothèque de l'Institut at Paris, and published by Ch. Ravaisson-Mollien in 1889.

† "Comincato ammilano addi 12 dissettembre 1508," as the manuscript states.

ity of the pyramid, the theory itself was developed by him, and in it he anticipated by half a century the labors of Maurolycus and Commandinus. Furthermore, as early as 1508, when Copernicus was only beginning his study of the solar system, Leonardo was already setting forth his views in opposition to the geocentric theory of the universe. In the same year, two generations before Palissy preached the doctrine, Leonardo formulated the principles that were afterwards accepted by the world as to the origin of fossils. In view of the testimony that M. Duhem adduces from the original manuscripts, M. Ravaisson's remark is not an exaggeration, that Leonardo should be looked upon as "le grand initiateur de la pensée moderne."

The second chapter treats of the indebtedness of the Spanish Jesuit Villalpand to the genius of Leonardo. Villalpand was born at Cordova in 1552 and died at Rome in 1608. The fact is established that Leonardo anticipated by a century certain discoveries of Galileo, that his knowledge of mechanics was remarkable for the time, and that he had apparently mastered all the works of his predecessors upon this subject. It is then shown that Villalpand's leading theorems upon the subject were apparently taken from or inspired by the manuscripts of Leonardo.

The third chapter is devoted to the work of Bernardino Baldi (1553–1617), one of the most versatile scholars of his age. M. Duhem shows that he drew very largely upon the writings of his predecessor, and particularly in his theories of vortices, of the center of gravity, and of mechanics in general.

Of still greater interest is the fourth chapter, in which the indebtedness of Roberval and Descartes is shown. This does not relate to pure mathematics, however, but only to questions of mechanics.

The fifth chapter relates to the works of Thimon (Temo Judæus, of the fourteenth century) and their influence upon Leonardo's theory of tides, and upon his general view concerning the laws of hydrostatics and hydrodynamics.

This is followed by a chapter upon the influence of Leonardo on Cardan (especially in De Subtilitate Libri XXI) and Bernard Palissy (in the theory of fossils). M. Duhem's conclusion as to Cardan is what we might expect: "Comme Villalpand, comme Bernardino Baldi, comme tant d'autres de ses contemporains, Cardan fut un plagiare; mais en plagiant les idées de

Léonard de Vinci, il les sauva de l'oubli ; grâce à la grande vogue de son livre étrange, il les sema partout, et son manque de scrupules leur fit produire les découvertes dont elles portaient le germe. Celui qui mène les pensées humaines fait servir au progrès de la Science les plus tristes faiblesses des savants."

The first volume closes with a chapter on the contributions of Leonardo to the theory of gravity, and one containing an excellent biography of Albert of Saxony.

The second volume, which appeared in 1909, three years after the first, contains four parts. The first of these relates to "les deux infinis," and discusses the theory of the infinite and the infinitesimal according to the ideas of Aristotle and the schoolmen, closing with extracts from Leonardo's notes upon the subject. The reader naturally looks to see if the latter give any evidence of anticipating Cavalieri, but if he looks with expectation he is disappointed, for Leonardo explicitly takes the opposite view of geometric magnitudes. No idea of an integral calculus shows itself; and since Leonardo's entire treatment relates to physics rather than to mathematics, one can hardly be surprised.

The second chapter relates to the question of the plurality of worlds. In this Leonardo combats the assertion of Albertus Magnus that such plurality is impossible. To such a position he was undoubtedly led by the works of Nicolaus von Cuss (Nicolaus Cusanus, Nicholas of Cusa, 1401–1464), to whose contributions the third chapter is devoted. This chapter contains one of the best biographies of this great savant of the fifteenth century to be found anywhere. It is particularly valuable as showing both the influences that made Nicolaus the scholar that he was, and those that his writings exerted upon Leonardo.

The work closes with the contributions of Leonardo to geology, already mentioned incidentally in the first volume.

To the study of pure mathematics Leonardo da Vinci cannot be said to have contributed in any serious way. We have, it is true, some interesting geometric proofs that are due to him, and it is well known that geometry appealed to him, but his bent of mind was rather towards mechanics and the underlying philosophy of applied mathematics than towards any form of analysis. It is in the theory of attraction, in hydrodynamics, in the beginnings of celestial mechanics, in the theory of perspective, and in related topics, that the contributions of Leonardo are to be found.

The work of M. Duhem is a monumental one and it is deserving of great commendation. He has made the learned world his debtor by this labor of love. He has not written a history of science, but he has composed a work of the kind that makes the history of science possible.

DAVID EUGENE SMITH.

The Fundamental Theorems of the Differential Calculus. By W. H. YOUNG. Cambridge University Press. ix + 72 pp.

As stated in the preface, "rigidity of proof and novelty of treatment have been aimed at rather than simplicity of presentation, though this has never been lightly sacrificed." The chapter headings are: I. Preliminary notions. II. Limits. III. Continuity and semicontinuity. IV. Differentiation. V. Indeterminate forms. VI. Maxima and minima. VII. The theorem of the mean. VIII. Partial differentiation and differentials. IX. Maxima and minima for more than one variable. X. Extensions of the theorem of the mean. XI. Implicit functions. XII. On the reversibility of the order of partial differentiation. XIII. Power series. XIV. Taylor's theorem.

The ϵ argumentation usually found in such books is entirely absent. The notion of a limit point of a set of points is taken for granted and by means of it the whole theory of limits is constructed. Infinity is included among points approached as a limit point and hence without further particular statement functions are permitted to approach infinity as a limit the same as any other value. Following Baire, functions are considered as approaching multiple limits instead of one unique limit. Hence it comes about that many theorems which we are wont to see stated for certain classes of functions in terms of the equality of unique limits are here stated for more general classes of functions in terms of the equality or inequality of the upper or lower limits approached. As an example we select the following:

If, as x approaches the values a_1, $f(x)$ and $F(x)$ have both the unique limit zero, or $+\infty$, or $-\infty$, then the limits of $f(x)/F(x)$ lie between* the upper and lower limits of $f'(x)/F'(x)$, provided

A. a is not a limiting point of common infinities of $f'(x)$ and $F'(x)$;

* x is said to lie between a and b if $a \leqq x \leqq b$.