The work of M. Duhem is a monumental one and it is deserving of great commendation. He has made the learned world his debtor by this labor of love. He has not written a history of science, but he has composed a work of the kind that makes the history of science possible.

DAVID EUGENE SMITH.

The Fundamental Theorems of the Differential Calculus. By W. H. YOUNG. Cambridge University Press. ix + 72 pp.

As stated in the preface, "rigidity of proof and novelty of treatment have been aimed at rather than simplicity of presentation, though this has never been lightly sacrificed." The chapter headings are: I. Preliminary notions. II. Limits. III. Continuity and semicontinuity. IV. Differentiation. V. Indeterminate forms. VI. Maxima and minima. VII. The theorem of the mean. VIII. Partial differentiation and differentials. IX. Maxima and minima for more than one variable. X. Extensions of the theorem of the mean. XI. Implicit functions. XII. On the reversibility of the order of partial differentiation. XIII. Power series. XIV. Taylor's theorem.

The ϵ argumentation usually found in such books is entirely absent. The notion of a limit point of a set of points is taken for granted and by means of it the whole theory of limits is constructed. Infinity is included among points approached as a limit point and hence without further particular statement functions are permitted to approach infinity as a limit the same as any other value. Following Baire, functions are considered as approaching multiple limits instead of one unique limit. Hence it comes about that many theorems which we are wont to see stated for certain classes of functions in terms of the equality of unique limits are here stated for more general classes of functions in terms of the equality or inequality of the upper or lower limits approached. As an example we select the following:

If, as x approaches the values a_1, $f(x)$ and $F(x)$ have both the unique limit zero, or $+\infty$, or $-\infty$, then the limits of $f(x)/F(x)$ lie between* the upper and lower limits of $f'(x)/F'(x)$, provided

A. a is not a limiting point of common infinities of $f'(x)$ and $F'(x)$;

* x is said to lie between a and b if $a \leqq x \leqq b$.

B. a is not a limiting point of zeros of $F'(x)$ unless these zeros are also zeros of $f'(x)$ and

C. $F(x)$ is monotone.

There are several departures from the usual treatments. Thus Baire's definition of upper and lower semicontinuity is changed in conformity with the definitions used by Young in the *Quarterly Journal*, volume 39, pages 67–83. The indeterminant forms $0/0$ and ∞/∞ are treated without the use of the mean value theorem, as is also Taylor's formula with a remainder.

In the chapter on Taylor's series the necessary and sufficient condition for the expansibility of a function of one real variable is stated to be that $y^{n+p} f^{(n)}_{(a+x)}/(n+p)!$ shall be bounded for all values of n, p, x (under certain restrictions). This seems to be a simplification of the well known condition of Pringsheim that the expression given must approach zero uniformly for certain y and x. It is curious that neither Pringsheim nor Young has hit upon the obvious condition, viz., that for some value of h, $h^n f^{(n)}_{(a+x)}/n!$ must be bounded or approach zero uniformly, as the case may be.

There is throughout an endeavor to make the treatment very general. This often makes the theorems clumsy and the proofs complicated. Necessary and sufficient conditions seem to be insisted on wherever such have been found.

The treatment not being designed for a first reading of the subject, many terms are left undefined, sometimes even to the confusion of the initiated. It is not surprising that semicontinuity (though upper and lower semicontinuity are defined) and uniform convergence of series are not defined, but it might well have been stated that a stretch means a rectangle.

Misprints have been noted as follows: page 7, 2d line, for *same* read *every*. Page 12, last line, insert exponent n. Page 32 in the statement of the theorem for $f^{(n)}(a+\theta h)/n!$, read $h^n f^{(n)}(a+\theta h)/n!$. Page 37, the "equation to zero" had better read the "equating, etc."

In several instances an interval is said to be closed when there is no need for such restriction (cf. the definition of continuity on page 8).

The reviewer believes greater simplicity as well as generality could be achieved in some instances by considering multiple valued functions as well as multiple valued limits. A possible further simplification of the condition for the expansibility of a function in Taylor's series has already been noted.

There are two appendices, one containing explanatory notes regarding certain theorems on sets of points which are involved in the treatment in the text and the other a short bibliography of recent papers by Young, Baire, Lebesgue, and others. Only twenty-five titles are given.

N. J. LENNES.

NOTES.

THE Eighteenth Summer Meeting of the AMERICAN MATHEMATICAL SOCIETY will be held at Vassar College on Tuesday and Wednesday, September 12–13. Titles and abstracts of papers intended for presentation at this meeting should be in the hands of the Secretary by Saturday, August 26.

THE April number (volume 12, number 3) of the *Annals of Mathematics* contains the following papers: "On the solutions of ordinary linear homogeneous differential equations of the third order," by G. D. BIRKHOFF; "Approximate representation," by W. E. BYERLY; "Note on cubic equations and congruences," by L. E. DICKSON.

AT the meeting of the London mathematical society held on April 27, the following papers were read: By G. T. BENNETT, "On the geometry of a deformable octahedron"; by W. P. MILNE, "A symmetrical method of apolarly generating cubic curves"; by G. N. WATSON, "The solution of the homogeneous linear difference equation of the second order (second paper)"; by G. B. MATHEWS, "A cartesian theory of complex geometrical elements of space"; by A. CUNNINGHAM, "The number of primes of given linear form"; by M. J. M. HILL, "On the proofs of the properties of Riemann's surfaces discovered by Lüroth and Clebsch."

At the meeting of May 11 the following papers were read: By G. T. BENNETT, "Exhibition of a model of a deformable octahedron"; by J. W. NICHOLSON, "The scattering of light by a large conducting sphere."

THE International commission on the teaching of mathematics will hold its meeting this year at Milan September 18–20 under the presidency of Professor F. KLEIN.

THE fourth international congress of philosophy was held at