and is not interested in the mere mathematics of the subject will probably find the book unsuitable.

Some misprints have been noted, but they seem to be few. The footnote on page 99, "A finite periodic function of the time must have the form A sin $(bt+\nu)$ or A cos $(bt+\nu)$, where A, b and ν are constants," is a decidedly curious statement.

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Die partiellen Differentialgleichungen der mathematischen Physik. Nach Riemann's Vorlesungen in fünfter Auflage bearbeitet von HEINRICH WEBER. Erster Band. Braunschweig, Vieweg und Sohn, 1910. xviii+528 pp. Unbound, M. 12; bound, M. 13.60.

THE earlier editions of this work have won for themselves an important place in every mathematical library, where they are frequently consulted both as reference books and for more consecutive reading supplementary to courses on mathematical physics. They have thus become so well known that an extensive review of the volume before us would be superfluous. Professor J. S. Ames has ably reviewed the fourth edition in this BULLETIN (volume 8, page 81). The following comment will therefore be limited to a comparison of the fourth and fifth editions.

In the preface to the fifth edition, Professor Weber remarks that some of the developments of mathematical physics during the past decade have been too significant to leave unmentioned, and that although they have not been carried sufficiently far to permit of finished exposition in a text-book, they should receive some attention in the pages to follow. He speaks of integral equations and the notion of relativity as two of the more important contributions made to the science since his last edition in 1900, and promises an application of the first in the present volume. The failure to find any satisfactory fulfillment of this promise will be the reader's greatest disappointment. In its place there appears a brief exposition of Neumann's method of the arithmetic mean, with the old assumption of a convex surface, a condition made unnecessary by the use of integral equations. A suggestion is indeed given for the removal of this restriction, but it is not developed further, and appears inadequate. It is not even pointed out that an integral equation is being solved, whereas, as Professor Mason has shown in his New Haven Colloquium lectures, Neumann's method admits a beautiful treatment in the garb of integral equations. A general outline of the theory of integral equations, sufficient for the purposes in hand, and quite in keeping with the character of the book, could have been given in a half dozen pages, and would have abbreviated the solution of the electrostatic problem which follows. A reference to Plemelj's monographs on the theory and its application to physical problems (*Monatshefte für Mathematik und Physik*, volume 15) would have been well in place.

The new edition has been freed from several errors occurring in the previous one. Two of these should be mentioned. The conditions that a function may be represented by a Fourier integral were incorrectly stated (4th edition, page 40). These have been corrected, and alternative conditions added. An example follows to which none of the criteria apply. It may be noted that Pringsheim has recently given broad conditions covering Weber's example (*Mathematische Annalen,* volume 68, pages 405–06). The expression for the potential energy of a pair of magnets (pages 366–67) has been corrected. A few minor errors have crept in, but their number is gratifyingly small. The definition of continuity of a function at infinity (5th edition, page 6), which characterizes the function as continuous in any finite subregion and as bounded, is unusual, to say the least.

The section on the gamma function will prove a welcome addition to the younger reader, and a number of references have been added at various points. This, however, is one respect in which greater improvement might have been made, since there has appeared recently a considerable number of excellent monographs and texts which would well supplement the briefer expositions of our author. The following additions appearing in the new edition have received from the major part of a section to a section apiece: Fourier's integral for several variables; the transformation of a differential expression of second order from one set of oblique coordinates to another; the definition of Bessel's functions of higher order in terms of the preceding ones; the expression of arbitrary functions in terms of integrals containing Bessel's functions; the solution in series of the general differential equation of Bessel; the expression for the divergence of a vector field in curvilinear coordinates; the values of the zonal harmonics for the values 0 and 1 of the argument. A whole division (Abschnitt) has been added on the general electrostatic problem; its first section deals with the general properties of Newtonian potentials and would seem to belong in an earlier division where these functions were first considered. The three remaining sections are devoted to the method of the arithmetic mean and Beer's treatment of the equilibrium of a charge on a conductor, with the example of the ellipsoid.

In general the author has been conservative in making changes. More valuable additions might have been made in some cases. Linear partial differential equations of second order, for instance, receive a scant two pages, which give only the device of finding particular solutions in the form of exponential functions with linear exponents. Undoubtedly the systematic development of integral equations and their applications, of the principle of relativity, and in all likelihood, of Lebesgue's theory of integration, will make necessary at some future time a book treating the same subjects in radically new ways. But for the present, and for some time to come, Weber's book, with its origins in Riemann's lectures, will continue to be indispensable. The new edition will find its way into the more complete libraries, although for smaller collections the previous one will prove wellnigh as useful, if a prediction can be made on the basis of the first volume of the new edition.

O. D. KELLOGG.

NOTES.

THE concluding (October) number of volume 34 of the *American Journal of Mathematics* contains the following papers: "The involutional birational transformation of the plane, of order 17," by VIRGIL SNYDER; "On the problem of two fixed centers and certain of its generalizations," by A. M. HILTEBEITEL; "Abstract definitions of all the substitution groups whose degrees do not exceed seven," by G. A. MILLER; "The attraction of a homogeneous spherical segment," by G. GREENHILL.

THE Prince Jablonowski society of Leipzig announces the following prize problem:

"What is the expression of the C. Neumann theorem (*Abhandlungen d. k. S. Gesellschaft d. Wissenschaften,* 1909) in the theory of the Newtonian potential, applied to the oval shaped