

An Introduction to the Lie Theory of One-Parameter Groups with Applications to the Solution of Differential Equations. By Dr. ABRAHAM COHEN. D. C. Heath and Company, 1911. vii + 248 pp.

THE theory of continuous groups has become a subject of such general importance, and constitutes a method of such great power, that every attempt to render it more easily accessible should be received gratefully. One of the advantages of the Lie theory is its elementary character, in so far as a large portion of it may be developed with very modest prerequisites. So much the more must we welcome a book which enables us to present this subject to undergraduate classes in our own language. It is always a serious thing to complicate matters for the younger students by adding linguistic to their inevitable mathematical difficulties. The large works of Engel and Scheffers on Lie's theory are written with admirable pedagogic insight, so that there would be little need of such a book as this in German. But the only larger English book on the subject, that by Campbell, is far from being a paragon of lucid exposition. The little book by Page is hardly extensive enough to serve the purpose of more than a mere introduction.

There is then a real field for such a book as Dr. Cohen's. We feel, however, as though the author had overemphasized the differential equation part of the theory. Still, that is a matter of taste.

The book is divided into seven chapters and an appendix. The first chapter naturally begins with the definition of a group of transformations, infinitesimal transformations, invariants, path curves, canonical variables, etc. In Chapter II these notions are applied to differential equations of the first order in connection with the theory of the integrating factor. The third chapter has the heading Miscellaneous theorems and geometrical applications, and deals among other things with isothermal curves. Chapter IV is on differential equations of the second and higher orders, and introduces the notion of n -times extended group. Chapter V is devoted to linear partial differential equations of the first order, while Chapter VI returns to ordinary differential equations of the second order. Chapter VII finally is on contact transformations. In the appendix are developed a number of the more difficult parts of the general theory, these matters being referred to in the

proper place in the main body of the text. Besides, the appendix contains two tables to serve as an aid in properly classifying and integrating differential equations.

The book is well written and is well adapted as a general introduction to Lie's theory. As already indicated, the too systematic treatment of differential equations from this point of view appears to the reviewer to be a defect. It is one thing to recognize that Lie's theory is capable of bringing some order into the haphazard methods of the elementary theory of differential equations; but it is quite another to use Lie's theory as a strait jacket which every differential equation must be made to fit.

And now for a few points of detail. The author's definition of a one-parameter group is rather clumsy, as a result of the unfortunately so common desire to put it all into one sentence. The two parts of remark 1 on page 7 are not obviously connected. The true reason for the validity of remark 1 is to be found in the latter part of remark 2. Although the author speaks everywhere of one-parameter groups, he really means such groups as are generated by an infinitesimal transformation. At least, a number of his theorems are not true for mixed groups. On page 176 in deriving the condition for a "union of lineal elements" he neglects the case where the union consists of the elements of a fixed point. The printing is well done. The reviewer has noticed only a few misprints. On page 45 last line read $-A\eta$ instead of $+A\eta$. Page 177 in Ex. 5 read $-\cot t$ instead of $-\cos t$. Page 188 just above equation (130') read characteristic instead of charactertisic.

A rather confusing feature is the use of two different scarcely distinguishable fonts for the same letters, as on page 24. There is no necessity for this. It seems a pity that the publishers have not seen fit to give the book a wider margin. The binding also is none too good.

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Abriss der Algebra der Logik. Bearbeitet im Auftrag der Deutschen Mathematiker-Vereinigung von Dr. EUGEN MÜLLER, Professor an der Oberrealschule zu Konstanz. In drei Teilen. Erster und Zweiter Teil. Leipzig und Berlin, B. G. Teubner, 1909-1910, 159 pp: 8 marks.

ALMOST from the days of Boole an ever-increasing need has been felt for an elementary introduction to the growing subject