I.

1 2 3	1 8 9	2 8 12	3 8 15	4 7 15	5 9 10	7 8 14
1 4 14	1 10 15	2 9 15	3 9 12	4 9 11	5 12 14	7 9 13
1 5 13	2 4 13	2 11 14	3 10 14	4 10 12	6 9 14	8 10 13
1 6 11	2 5 7	3 4 5	3 11 13	5 6 15	6 12 13	11 12 15
1 7 12	2 6 10	3 6 7	4 6 8	5 8 11	7 10 11	13 14 15

II.

1 2 5	1 9 13	2 7 14	3 6 15	4 8 10	5 13 14	8 9 14
1 3 8	1 11 12	2 8 13	3 11 13	4 11 14	6 7 9	9 10 11
1 4 15	2 3 9	2 11 15	3 12 14	5 6 11	6 8 12	9 12 15
1 6 14	2 4 12	3 4 7	4 5 9	5 7 12	7 8 11	10 12 13
1 7 10	2 6 10	3 5 10	4 6 13	5 8 15	7 13 15	10 14 15

The group for each of the above triple-systems is of order 3 and is generated by the substitution

$$s = (1\ 7\ 14)\ (2\ 6\ 10)\ (3)\ (4\ 12\ 8)\ (5\ 9\ 15)\ (11)\ (13).$$

February, 1913.

DE SÉGUIER'S THEORY OF SUBSTITUTION GROUPS.

Théorie des Groupes finis. Eléments de la Théorie des Groupes de Substitutions. Par J. A. DE SÉGUIER. Paris, Gauthier-Villars, 1912. x+228 pp. 10 Fr.

THE earliest separate book on the theory of groups is Jordan's Traité des Substitutions, which appeared in 1870 and is still one of the most valuable treatises along certain lines. Twelve years later there appeared Netto's Substitutionentheorie, which was translated into Italian by G. Battaglini, in 1885, and into English by F. N. Cole, in 1892. Five years after this English translation, W. Burnside published the first separate treatise on groups originally written in our language, under the title Theory of Groups of Finite Order. A second and enlarged edition of this work appeared in 1911. In 1900 the first printed edition of a book on this subject originally written in Italian appeared under the title Lezioni sulla Teoria dei Gruppi di Sostituzioni, by L. Bianchi.

Since the beginning of the present century new books on the theory of groups of finite order have appeared more rapidly, as may be seen from the following list: L. E. Dickson, Linear Groups, 1901; J. A. de Séguier, Groupes abstraits, 1904; H. Hilton, Introduction to the Theory of Groups of Finite Order, 1908; E. Netto, Gruppen und Substitutionentheorie, 1908;

J. A. de Séguier, Eléments de la Théorie des Groupes de Substitutions, 1912. Among the other works which have appeared during the same period, and which tend to make this theory more easily accessible, we may mention the extensive articles in the Encyclopédie des Sciences mathématiques and in Pascal's Repertorium, and the bibliographies published by B. S. Easton in 1902 and by C. Alasia in 1908–1909.

The book under review is largely based on the Groupes abstraits by the same author, which was reviewed by L. E. Dickson in this BULLETIN, volume 11 (1904–1905), pages 159–162. The closing lines of this review are as follows: " The reader of the present volume will be impressed with the author's complete mastery of his subject and will find in it a useful compact summary of the results to date in the purely abstract part of finite group theory." A similar general verdict applies to the volume before us. Although this volume is somewhat larger than its predecessor, it is written in the same compact style and contains a very large amount of information. Many readers would doubtless prefer a less compact form and a less extensive use of special symbols; but after these symbols have been mastered, they tend towards clearness as well as towards brevity.

Just as in the Groupes abstraits the number of minor errors in the present volume is large. Lists of such errors together with some complements relating to both of these volumes appear near the end of the present volume. These lists under the heading " Additions et Corrections " cover eleven closely printed pages. It is evident that such a multitude of minor errors must be a source of much confusion to those who are not fully familiar with the subjects treated. In fact, the reviewer feels that both of these volumes are better suited for the student who has already arrived at considerable maturity in the theory of groups than for the beginner. The tendencies to begin with the general instead of the special cases and to express the results very concisely have many advantages but they present too many difficulties for most beginners.

As a work of reference the present volume offers some advantages over its predecessor since it contains an " Index des terms " and an " Index des notations." Together these two indices cover only two pages, and there is no subject index. In view of the fact that many of the most modern developments are considered, an author index would also have rendered useful service in several ways, There are no lists of exercises, but numerous illustrative examples are given, and many references, especially to the Groupes abstraits, are found in the text.

The volume consists of five chapters bearing the following headings in order: Substitutions; Groupes de substitutions, théorèmes généraux; Representation des groupes par des groupes de substitutions; Groupes de degré n et de classe $n-1$, groupes linéaires; and Groupes de degré kp, $p+\alpha$, $2p+\alpha$. These five chapters are followed by three "notes" covering 45 pages, and bearing the following headings: Sur la théorie des matrices, Equations algébriques, and Sur les groupes de degré n et de classe $n-1$. The second of these "notes" is the longest, and it treats, in a concise form, the general Galois theory of algebraic equations.

The concept of substitution is based upon a (1, 1) correspondence between the elements of a set (ensemble), and a substitution is said to consist of the operation of replacing an element by the corresponding element in such an automorphism. A substitution is said to be normal if the cycles are separated by periods. For instance, the substitution $(abcf)(dgh)(ke)$ may be represented in a normal form as follows: $abcf \cdot dgk \cdot ke$. This form has been employed by several other writers and it appears to the reviewer as the most convenient notation for substitutions in the restricted sense.

The fact that the technical terms are not always defined before they are employed in the present volume may be due to a desire to avoid, as far as possible, the repetition of definitions given in the Groupes abstraits. As instances of this fact, we may state that the technical terms normale and isomorphes are used on page 4, but these terms have not been defined on the preceding pages. The latter part of the first chapter is devoted to a consideration of polynomials representing substitutions in a Galois field, and it naturally contains a number of results due to L. E. Dickson.

The second chapter begins with a consideration of some of the fundamental properties of imprimitive and of intransitive groups. It is observed that the substitutions of each transitive constituent of an intransitive group must constitute a group, and these constituents are then combined into two sets denoted by A and B respectively. It is asserted, on page 28, that each of the factor groups of the entire group is a factor group of A or of B. This statement is evidently incorrect since the entire group may have factor groups whose orders exceed the order of each of the constituent groups A and B. The chapter closes with a consideration of relations between the class, degree, and order of a group.

The proposition that every abstract group can be represented in an infinite number of different ways as a substitution group constitutes the opening sentence of Chapter III. Instead of pursuing the usual method of first proving that every finite group can be represented as a regular substitution group, our author starts with the general proposition and establishes it directly. An abstract group is said to be represented properly by a given substitution group if there is a (1, 1) correspondence between these groups; and the representation is said to be of the first, second, or third species as the augmented co-sets are multiplied on the right or left, or are transformed by all the operators of the abstract group, to obtain the substitutions of the representing group. Different simple isomorphisms between an abstract and a substitution group which represents it properly are said to be different representations of this group.

A footnote on page 86 is devoted to a consideration of an erroneous theorem announced by A. L. Cauchy as regards simply transitive primitive substitution groups. According to this theorem a primitive group of degree n cannot be simply transitive unless $n - 1$ is divisible by the degree of each transitive constituent of the subgroup composed of all the substitutions which omit a given letter. Hence a primitive group of degree $p + 1$, p being an odd prime, could not be simply transitive. The author of the present work calls attention to the facts that the first part of this theorem is easily seen to be incorrect and that the latter part has been verified for $p \leqq 13$ by C. Jordan and others, but he does not state that for $p = 83$ it has been proved that this part is also incorrect. Cf. G. A. Miller, *Bibliotheca Mathematica,* series 3, volume 10, 1910, page 321.

Chapter IV begins with a consideration of the transitive substitution groups of degree n in which all the substitutions except identity involve at least $n - 1$ letters. The greater part of the chapter is, however, devoted to a study of linear groups. A number of abstract definitions of well-known groups are incidentally developed, and simple isomorphisms between various groups are investigated. The chapter closes with a determination of the groups which can be represented on a prime number p of letters and which involve exactly $p + 1$ subgroups of order p.

The last chapter starts with a proof of the important theorem due to Burnside, which states that a transitive group on p letters must be multiply transitive whenever it involves more than one subgroup of order p. The proof is based upon the one given by I. Schur in the *Jahresbericht der Deutschen Mathematiker-Vereinigung*, volume 17 (1908), page 171. This is followed by a study of the interesting theorems relating to the multiply transitive groups of degree $p + \alpha$ which involve subgroups of order p. The latter part of the chapter is devoted to a study of the well known multiply transitive groups due to Mathieu.

In the preface we are told that the present volume is devoted to the substitutions which may be called natural, that is, to the substitutions on a finite number of objects whose order is simple. The author enters only partly into the field of linear modular groups. A more profound study of these groups, and a determination of systems of solvable groups, constitute the subjects of a proposed later volume by the same author. It is to be hoped that this later volume will contain at least a subject index, including the material of the present volume and of the earlier volume on Groupes abstraits. Such an index would make these volumes much more valuable for reference. A general author index would also render useful service.

G. A. MILLER.

WILSON'S ADVANCED CALCULUS.

Advanced Calculus: A Text upon Select Parts of Differential Calculus, Differential Equations, Integral Calculus, Theory of Functions, with Numerous Exercises. By EDWIN BIDWELL WILSON. Boston, Ginn and Company, 1912. ix+566 pp.

SOME years ago Professor Asaph Hall, after reading carefully Poincaré's Mécanique Céleste, which had just been published, wrote to its distinguished author and took him severely to task because he had devoted his splendid mathematical knowledge