known groups are incidentally developed, and simple isomorphisms between various groups are investigated. The chapter closes with a determination of the groups which can be represented on a prime number p of letters and which involve exactly $p + 1$ subgroups of order p.

The last chapter starts with a proof of the important theorem due to Burnside, which states that a transitive group on p letters must be multiply transitive whenever it involves more than one subgroup of order p. The proof is based upon the one given by I. Schur in the *Jahresbericht der Deutschen Mathematiker-Vereinigung,* volume 17 (1908), page 171. This is followed by a study of the interesting theorems relating to the multiply transitive groups of degree $p + \alpha$ which involve subgroups of order p. The latter part of the chapter is devoted to a study of the well known multiply transitive groups due to Mathieu.

In the preface we are told that the present volume is devoted to the substitutions which may be called natural, that is, to the substitutions on a finite number of objects whose order is simple. The author enters only partly into the field of linear modular groups. A more profound study of these groups, and a determination of systems of solvable groups, constitute the subjects of a proposed later volume by the same author. It is to be hoped that this later volume will contain at least a subject index, including the material of the present volume and of the earlier volume on Groupes abstraits. Such an index would make these volumes much more valuable for reference. A general author index would also render useful service.

G. A. Miller.

WILSON'S ADVANCED CALCULUS.

Advanced Calculus: A Text upon Select Parts of Differential Calculus, Differential Equations, Integral Calculus, Theory of Functions, with Numerous Exercises. By Edwin Bidwell Wilson. Boston, Ginn and Company, 1912. ix+566 pp.

Some years ago Professor Asaph Hall, after reading carefully Poincaré's Mécanique Céleste, which had just been published, wrote to its distinguished author and took him severely to task because he had devoted his splendid mathematical knowledge and ability not to the advancement of astronomical science or to the invention of improved methods of investigation, but to mere criticism.

One of my friends, a brilliant physicist and a learned and skilful mathematician, has frequently said concerning the "exact mathematics" of our day that what we call "rigor" will be regarded by our descendants as we regard the rigor of our ancestors, an opinion by the way very strongly hinted at by Professor Bôcher in his St. Louis address.

We live in a critical age. Our present mathematical ideal is to draw from a carefully framed set of definitions and postulates by what we consider rigorously logical reasoning conclusions which seem to us necessary.

In addition to critical mathematicians, pure logicians to whom intuitive methods are like a red rag to a bull, there have always been creative mathematicians, often careless fellows who had a surprising knack for getting correct results by methods quite open to criticism; and mathematical artisans,—astronomers, physicists, engineers—to whom mathematics is a tool, and who are much more interested in results than in methods.

Far be it from me to disparage the critic; he is a useful person even if he is apt to be a little intolerant; but let us not train all our promising young men as if criticism were the only thing or even the main thing to work toward. Perhaps some of them may develop creative power; certainly many of them will become mathematical artisans; and life is short.

In our teaching let us by no means disregard the modern methods and the modern spirit. Let us familiarize the student with the present notions of rigor and with the received critical logical methods, but let us not stop there. Let us train him to use intuitive methods freely and to cultivate his powers of invention, and in case of need to work rapidly and even recklessly. If properly taught he will be in position to check his results and processes if they are challenged or if he doubts their accuracy, and yet he will have gained the power and confidence that will carry him forward swiftly and in the main safely. Better an occasional mistake through over-confidence than a perfectly safe snail's pace over ground every foot of which he has carefully tested before he has dared trust it with his weight.

That this is the ideal toward which the author of the book under review has striven is clear from his preface. "It has been fully recognized that for the student of mathematics the work on advanced calculus falls in a period of transition—of adolescence—in which he must grow from close reliance upon his book to a large reliance upon himself. Moreover, as a course in advanced calculus is the ultima Thule of the mathematical voyages of most students of physics and engineering, it is appropriate that the text placed in the hands of those who seek that goal should by its method cultivate in them the attitude of courageous explorers, and in its extent supply not only their immediate needs, but much that may be useful for later reference and independent study.

With the large necessities of the physicist and the growing requirements of the engineer, it is inevitable that the great majority of our students of calculus should need to use their mathematics rapidly and vigorously rather than with hesitation and rigor. Hence, although due attention has been paid to modern questions of rigor, the chief desire has been to confirm and to extend the student's working knowledge of those great algorisms of mathematics which are naturally associated with the calculus." In my opinion the ideal is an admirable one; it seems to me that the author's approximation to that ideal has been a remarkably close one.

The first chapter, Review of fundamental rules, contains a masterly sketch of the groundwork of the calculus, differential and integral, with which the student is supposed to be familiar when he begins the study of the book, and serves as a guide and as an aid to him in the careful review of his elementary course which perforce he must make before going on to the higher parts of his subject. In this chapter the proofs are often merely hinted at, or if given are put in the briefest possible form without any attempt to avoid the use of intuition or to strive at modern rigor.

The second chapter, Review of fundamental theory, is a chapter in "exact mathematics." In the words of the author "the object of the chapter is to set forth systematically, with attention to precision of statement and accuracy of proof, those fundamental definitions and theorems which lie at the basis of calculus, and which have been given in the previous chapter from an intuitive rather than a critical point of view."

It is needless to say that for most students beginning advanced calculus this chapter is in no sense a review. It is new work and hard work. Such matters as the Concept and theory of real number (very briefly set forth); Definition of a limit; Theorems on limits and on sets of points; Real functions of a real variable; Continuity; Uniform continuity; Differentiability; Rolle's theorem and the theorem of the mean; Summation and integration; Integrability, are taken up from the modern point of view and the modern rigorous theories and proofs are carefully and well given.

During the rest of the book references are freely made to this chapter, and occasionally an important or fundamental proof is put into modern rigorous form, but in the main there is a refreshing absence of epsilons and deltas and the rest of the paraphernalia of the critical mathematician.

As an avowed treatise on advanced calculus the book begins with Chapter III, and is almost encyclopædic in its range. Topics treated exhaustively, topics briefly sketched, topics merely hinted at and illustrated or suggested by problems chosen from the fields of pure analysis, of mechanics, of engineering, and of physics are almost without number, and are by no means fully revealed by the excellent table of contents, or even by the uncommonly detailed index.

To the teacher or to the working mathematician the work is invaluable. It probably was not written for the unaided student. He would certainly find it too condensed and too difficult. In the hands of a skilful teacher it might be an effective text book, but even then the class would probably find it rather hard sledding.

The labor of preparing the book must have been enormous and the author deserves the thanks of the mathematical public for a most valuable addition to the literature of the calculus.

W. E. Byerly.

THE CALCULUS IN INDIA.

A Text-book of Differential Calculus. By G. Prasad. Longmans, Green and Co., 1909. xii+161 pp.

A Text-book of Integral Calculus. By G. Prasad. Longmans, Green and Co., 1910. x+241 pp.

Twin texts on calculus from Benares, Holy City of the Hindus! If introduced in this country they would be pro-