work and hard work. Such matters as the Concept and theory of real number (very briefly set forth); Definition of a limit; Theorems on limits and on sets of points; Real functions of a real variable; Continuity; Uniform continuity; Differentiability; Rolle's theorem and the theorem of the mean; Summation and integration; Integrability, are taken up from the modern point of view and the modern rigorous theories and proofs are carefully and well given.

During the rest of the book references are freely made to this chapter, and occasionally an important or fundamental proof is put into modern rigorous form, but in the main there is a refreshing absence of epsilons and deltas and the rest of the paraphernalia of the critical mathematician.

As an avowed treatise on advanced calculus the book begins with Chapter III, and is almost encyclopædic in its range. Topics treated exhaustively, topics briefly sketched, topics merely hinted at and illustrated or suggested by problems chosen from the fields of pure analysis, of mechanics, of engineering, and of physics are almost without number, and are by no means fully revealed by the excellent table of contents, or even by the uncommonly detailed index.

To the teacher or to the working mathematician the work is invaluable. It probably was not written for the unaided student. He would certainly find it too condensed and too difficult. In the hands of a skilful teacher it might be an effective text book, but even then the class would probably find it rather hard sledding.

The labor of preparing the book must have been enormous and the author deserves the thanks of the mathematical public for a most valuable addition to the literature of the calculus.

W. E. Byerly.

THE CALCULUS IN INDIA.

A Text-book of Differential Calculus. By G. Prasad. Longmans, Green and Co., 1909. xii+161 pp.

A Text-book of Integral Calculus. By G. Prasad. Longmans, Green and Co., 1910. x+241 pp.

Twin texts on calculus from Benares, Holy City of the Hindus! If introduced in this country they would be productive, we fear, of very unholy comments by the students; for they are hard and mathematical, and pay but small respects to the feeble intellect that longs for a nurse and for practical applications, in short for being entertained, from the start. Yet these texts have most excellent characteristics, chief among which from our point of view is their difference from the ordinary run of texts. The volumes are "intended for beginners and so designed as to meet the requirements of Part I of the Cambridge Mathematical Tripos Examinations, and of the Examinations for the B.A. and B.Sc. degrees of Indian Universities."

The author has gone to the limit in abolishing the infinitesimal and differential from both differential and integral calculus. At first we thought that there was no mention at all of infinitesimals, but finally we found, the last thing in the Differential Calculus, in fine type, as a footnote to the chapter on indeterminate forms, the definitions of infinitesimal and infinites, with a statement that they were not of much importance to the beginner. So far the definition of differential has proved elusive, though the symbol is used in d/dx and in $\int dx$. The idea of a limit of an infinite sum of infinitesimals is relegated to a short chapter by itself, starred as difficult, and is not used in the applications; alas, poor Duhamel. But why not? In many recent texts on calculus the infinitesimal and differential are, for the sake of precision, bereft of all their fecundity and of all their effectiveness and naturalness as a method of thought for the student. When thus slighted it is as well to discard them utterly.

The two volumes together, apart from appendices and answers, total 345 pages. The pages are wide open, plenty of unused paper, scarcely a useless word. The text is therefore really short. And this is the more noteworthy in view of the amount of space the author will give to rubbing in some fundamental process. Thus in most books when $\sin x$ has been differentiated, the formulas for differentiating other trigonometric functions and the inverse functions are derived by various devices. Not so with Prasad; he takes each one and determines ab initio the limit of $[f(x+h) - f(x)]/h$. He then gives exercises where the student may do likewise, and it is not until the following chapter that he develops the rules for sums, products, quotients, functions of functions, and the like. There are numerous and varied exercises upon which the student may practice formal differentiation.

Chapter IV deals with tangents and normals in rectangular and polar coördinates, and such allied questions as subnormals. The author derives the equation of the tangent and appends two notes, in the first of which he remarks that the derivative is the slope of the tangent, and in the second derives the expression for ds/dx. The method is solely one of limits; personally we have a preference for infinitesimal figures which show such formulas at a glance and afford a visual means of remembering them. A chapter on asymptotes follows, and then one on curvature. The center of curvature is defined as the limiting position of the intersection of a fixed normal and a variable normal approaching the fixed one as a limit. Thus the center of curvature is found first, the radius subsequently. The treatment extends to polar coördinates, to involutes and evolutes, and to concavity, convexity, and points of inflection. There is a brief chapter on envelopes and a long one on curve tracing and on properties of special curves.

Although the symbol for the second derivative has been defined and used in connection with curvature, the author comes only now to his chapter on successive differentiation, and he makes it a real chapter, not a mere note with silly exercises. He finds the nth derivative of x^m, a^x, $\sin x$, $\cos x$, $e^{ax} \sin bx$, $e^{ax} \cos bx$, of a rational fraction (by decomposition into partial fractions), of products (Leibniz's formula), and of various functions for which a recurrence formula may be established. The good student will learn a lot out of this chapter, the poor student, we fear, will be unable to do anything—although the author states that these texts are based upon his experience in teaching a large number of pupils. We are now ready for Rolle's theorem and the finite or infinite developments of functions by Taylor's or Maclaurin's theorems; and the careful work on successive differentiation enables us to expand a great many functions—$e^{a \sin^{-1} x}$, for example—other than the common elementary functions. The volume closes with a brief account of maxima and minima, and indeterminate forms. There are short notes on Weierstrass's continuous non-differentiable function, on Rolle's and Taylor's theorems, and on partial differentiation.

The Integral Calculus begins with the definition of the indefinite integral, and of the definite integral as the difference of the indefinite between limits—a dangerous definition, as it makes the integral of $1/x^2$ from -1 to $+1$ equal to -2. It is stated that the definite integral is a limit of a sum, and the reader is left to verify the fact by calculating the limit for some simple functions and comparing it with the value obtained by integration. Then follows a long chapter on fundamental methods of integration—integration by substitution, by parts, and by reduction formulas. The author does not use the differential method as

$$\int \frac{dx}{x \log x} = \int \frac{d \log x}{\log x} = \log \log x,$$

but the method of substitution as $x = \varphi(t) = e^t$, $\varphi'(t) = e^t$,

$$\int f(x)dx = \int f(x) \frac{dx}{dt} dt = \int \frac{1}{t} dt = \log t = \log \log x.$$

Moreover, he uses the formula for integration by parts as

$$\int uvdx = uV - \int u'Vdx,$$

where V is the integral of v.

For those who are never to separate the derivative into its differentials and those who make the separation only at that late stage when the student is beginning integration, and has enough difficulties with integration alone without having a new notation for differentiation, this method is to be recommended. Indeed the author might more logically have used D for differentiation (and D_t when the differentiation was performed with respect to some variable t other than the apparent one) and $\int$ without the dx for integration ($\int_t$ when the variable is t). We believe, however, that the differential method is better, and hope it has suffered only a temporary total or partial eclipse. We believe that Huntington, in his syllabus of calculus for the Society for the Promotion of Engineering Education, has cast a line to the process of differentiation, as contrasted with derivation; we trust he may rescue and resuscitate it.

Prasad next gives a systematic treatment to the integration of algebraic rational and irrational fractions. This is starred as more difficult and possible of omission in the first reading. Then follows a chapter on integrating transcendental functions. In the next chapter we come to definite integrals, their definition as the limit of a sum, their general properties, and their evaluation in a few simple cases. The work thus far covers a little more than half the volume; the remaining portion is given to various applications.

The remarkable thing about all these applications is the complete omission of any ideas concerning limits of sums. The method is always to find a derivative, and then integrate. For example, the result for ds/dx has been found; hence s may be obtained. In like manner $dA/dx = y$ may be established, and hence A is the integral of y. And so on, to arcs and areas in polar coördinates, to surfaces and volumes of revolution, to centers of gravity, centers of pressure, moments of inertia, and attractions. All are treated by differentiation. Why not? Why not eliminate the troubles connected with limits of sums? The author has made the presentation clear and rigorous, and has shown conclusively that we do not need to bother with the integral as a limit of a sum in elementary calculus. His method is worthy of our most serious consideration—if we desire to be rigorous instead of suggestive, and we can hardly be both in a first course on calculus.

The remaining applications are to the dynamics of a particle, prefaced by a few sections on the integration of the simpler differential equations. There are notes on the integration of infinite series, on Riemann's discontinuous integrable function, and on Fourier's series.

These texts merit our special consideration because they are different from those we are used to. It would be interesting to see them tried on American classes both for the effect on the students and for the effect on the teachers.

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SHORTER NOTICES.

Mêmoires Scientifiques. By PAUL TANNERY. Publiés par J. L. HEIBERG and H. G. ZEUTHEN. I. *Sciences Exactes dans l'Antiquité*, 1876–1884. Toulouse, Edouard Privat; Paris, Gauthier-Villars, 1912. xix+465 pp. Price 15 francs.

IN our time there have been three men whose love for ancient science and whose perfect command of the Greek language