

curve. Then follows a systematic study of the shortest distances associated with two neighboring trihedrals. From convenient forms for the two principal formulas for the shortest distance between two skew lines are deduced the shortest distances (1) between two neighboring positions of  $g$ , (2) between a point  $P$  of the space curve and the line of striction of the ruled surfaces formed by the lines  $g$ , and (3) between  $P$  and the "normal plane" of a neighboring point on the curve. Six other formulas are obtained by substituting for  $g$  the binomial and the principal normal. The relations among these nine distances become noteworthy when the general trihedral is replaced by the canonical. Another special case is that in which  $g$  is required to be perpendicular to the tangent to the curve. This case becomes more important when the further restrictions are imposed that the curve shall lie on a certain surface and that  $g$  shall fall along the positive direction of the normal to the surface at the point. An appendix, which occupies one-third of the book, deals with generalized curvature in  $R_n$  and with the invariant representation in  $R_3$ .

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*Grundlagen der Geometrie.* Von DAVID HILBERT. Fourth edition. Leipzig, Teubner, 1913. vi+258 pp.

THE system of axioms of geometry presented in this new edition of Hilbert's classical work differs from that given in the third edition (1909) in regard to the third group of axioms, those of congruence, the number of which is now reduced from six to five. In the third edition, axiom III: 5 reads: When an angle ( $h, k$ ) is congruent to both the angle ( $h', k'$ ) and the angle ( $h'', k''$ ), then the angle ( $h', k'$ ) is congruent to the angle ( $h'', k''$ ). In the present edition, this is now stated as theorem 10, and for a proof based upon the axioms in groups I and II and the remaining five axioms in group III, the reader is referred to a paper by A. Rosenthal: "Vereinfachungen des Hilbertschen Systems der Kongruenzaxiome," *Mathematische Annalen*, volume 71 (1912), pages 257-274.

A few other, but relatively unimportant, changes have been made in the text, and references to recent literature have been added.

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