that both $\epsilon(x)\log x$ and $\epsilon(x)\log w(x)$ are increasing functions. But if $\log\log w(x)$ is made to increase by only a finite amount outside of a set of intervals $(x_i, x_i + \delta_i)$, $i = 1, 2, \cdots$, and if these intervals be so chosen that the series

$$\frac{\delta_1}{x_1} + \frac{\delta_2}{x_2} + \cdots$$

converges, it is easy to see that $\epsilon(x)$ cannot approach zero. The second proof (note I, pages 117–128) is however devoid of any objection.

With the aid of the concept of function-type Blumenthal generalizes almost all the results known for entire functions of finite order but it is perhaps not desirable that the reviewer make an outline of this material.

Here then is a book which the mathematician who is interested in the theory of the entire function will find worthy of his attention. GEORGE D. BIRKHOFF.

Les Systèmes d'Equations linéaires a une Infinité d'Inconnues. Par FRÉDÉRIC RIESZ. Paris, Gauthier-Villars, 1913. vi + 182 pages. 6.50 fr.

THIS little book belongs to the collection of monographs on the theory of functions published under the general direction of M. Emile Borel. It deserves the high praise of being pronounced worthy a place in this excellent series.

The purpose of the volume is to give a rapid exposition of the fundamental ideas, of the methods and of the principal results in the theory of linear equations with an infinite number of variables—a theory which is due almost entirely to contemporary mathematicians.

An introduction to the subject is made through a chapter (Chapter I, pages 1–20) devoted to the beginnings of the theory. The method of undetermined coefficients was the first; but this method is not characteristic of the subject. Next comes the work of Fourier, who introduced an important general principle in connection with a special type of problem: naturally, the work of Fourier does not meet the modern requirements of rigor. Mention is also made of the papers of Fürstenau and Kötteritzsch; these are said to have been without importance in the development of the theory. (In this connection see a paper by the reviewer in the *American Journal of Mathematics* for January, 1914.) Finally, an account of the work of Appell is given in the latter part of the first chapter.

Chapter II (pages 21–41) begins with a brief account of the ideas introduced by the American astronomer, Dr. G. W. Hill, in his very important memoir on the motion of the lunar perigee. This memoir was the starting point for the modern theory of infinite determinants and of linear equations with an infinite number of variables. Poincaré first supplied the convergence proofs which are needed to justify the processes used formally by Hill. With the ideas of Hill and Poincaré as a basis, a theory of considerable extent and wide range of applicability has been built up, principally by H. von Koch. An account of the general elements of this theory, in their simplest form and without applications, is given in Chapter II.

Up to this point the author has followed the chronological order, except that (in Chapter I) he has given an account of the work of Appell before that of Hill. He now inverts the chronological order, for greater convenience in exposition, giving an account (in Chapter III, pages 42–77) of Schmidt's far-reaching theory before taking up the earlier work of Hilbert. This theory of Schmidt, the most general hitherto developed, is itself extended by the present author so as to include a somewhat larger range of results than those obtained by Schmidt in his original account.

In connection with this chapter one would do well to read the elegant paper by Bôcher and Brand in the *Annals of Mathematics* (2), 13 (1912): 167–186, where an exposition of the Schmidt theory is given which leaves nothing to be desired from the point of view of the reader who is forming a first acquaintance with the subject. In order to have a deep understanding of the theory the reader should also see the geometric interpretation given to the whole matter in the original presentation by Schmidt. This latter paper is fundamentally so simple and elegant that it is sure to become one of the classics of mathematical literature. It is to be regretted that the book under review does not contain an exposition of the geometric ideas lying at the basis of this theory of Schmidt. By means of these geometric considerations the whole matter receives an illumination which probably can be procured for it in no other way.

Chapters IV (pages 78–121) and V (pages 122–155) are devoted to the theory of Hilbert as expounded by him in his now famous memoirs on the theory of integral equations and by some of his disciples in subsequent contributions. The first of these two chapters is given to the theory of linear substitutions where the number of variables is infinite. The general theory which is developed from this point of view is applied to the derivation of important results concerning a certain type of system of equations with an infinite number of variables. The second of these two chapters is given to the theory of quadratic forms where the number of variables is infinite and to the application of this theory to that of linear equations.

Chapter VI (pages 156–180) is devoted to certain applications of the general theory developed in the preceding part of the book. It falls into three parts dealing with as many distinct topics; namely, linear differential equations in which the coefficients are expansible in Laurent series, integral equations and trigonometric series.

This little book will serve a useful end by affording a ready introduction to one of the most important and most readily accessible phases of the general theory of functions of an infinite number of variables, a field in which at present there lies out before us a vast domain of unexplored territory—a domain in which the present generation will probably make further important explorations.

R. D. CARMICHAEL.

A General Course of Pure Mathematics from Indices to Solid Analytical Geometry. By ARTHUR L. BOWLEY, Sc.D. Oxford, Clarendon Press, 1913. xii + 272 pp.

It would be difficult to give a better brief account of the contents and the purpose of this book than that supplied by the author in the preface. From his remarks, therefore, we shall quote a few sentences, as follows:

"This book is the result of an attempt to bring within two covers a wide region of pure mathematics. Knowledge is assumed of that part of mathematics usually required for matriculation, namely algebra to simultaneous quadratic equations and the substance of the first four books of Euclid, together with a very slight acquaintance with graphic algebra, mensuration, and solid geometry. From this stage the work is carried forward in algebra to the logarithmic series; in