

*The Pell Equation.* A history of the equation  $x^2 - Ay^2 = 1$ , table of solutions from  $A = 1,501$  to  $A = 1,700$ , bibliography, table of continued fractions for  $\sqrt{A}$ . By EDWARD EVERETT WHITFORD. Lancaster, Pa., The New Era Printing Company, 1912. iv+193 pp.

THIS book presents in a very readable form an historical account of this famous problem. The author first discusses, in considerable detail, various efforts of the ancient Greek and Hindu mathematicians to find rational approximations to square roots, showing how these ultimately depend upon solutions of special cases of the Pell equation.

While these efforts and those of later mathematicians to find square root approximations do form the historical background of this problem, its formulation in general terms was made by Fermat in 1657. The author discusses the contributions of Lord Brouncker and of Euler toward the solution of the general case and justly credits Lagrange with the first rigorous proof of its solvability.

The relation of the Pell equation to the theory of quadratic forms is shown in connection with the more modern methods of Gauss and Dirichlet.

The excellent bibliography, together with the tabulated continued fraction developments of  $\sqrt{A}$  from  $A = 1,501$  to  $A = 2,012$ , as well as the table of fundamental solutions of the Pell equation as indicated in the sub-title, make this a very useful book for the worker in this field.

T. M. PUTNAM.

*Die Berührungstransformationen: Geschichte und Invariantentheorie.* Zwei Referate, der deutschen Mathematiker-Vereinigung erstattet von H. LIEBMANN und F. ENGEL. (*Jahresbericht der deutschen Mathematiker-Vereinigung. Der Ergänzungsbände V. Band.*) Leipzig, Teubner, 1914. v + 79 pp.

IN the first of these reports (pages 1-14), Liebmann gives an attractive sketch of the historical development of the theory of contact transformations. The second report (pages 15-77) on "Lie's theory of invariants of contact transformations, and its extension" by Engel is an original contribution to the theory in question rather than a report. As Engel points out in his introductory remarks, Lie took