

(pages 65–71), the use of logarithmic paper, and the proper treatment of physical data.

Typographical errors are numerous. Among other errors “the trajectory of the projectile of a German army bullet” (page 396) is particularly offensive. The statement (page 214) that the principle of logarithms “had been quite overlooked by mathematicians for many generations” is not correct, for the principle was known even to Archimedes and appeared and was discussed in books of the sixteenth century. The development of negative, fractional, and irrational numbers (page 355) is the logical one, and not from “the history or algebra.” In the treatment of trigonometry the constant use of all six trigonometric functions would seem to be open to criticism. There appears also repeated emphasis upon rather trivial schemes for memorizing formulas and even the signs of ordinate and abscissa (or of $\sin \alpha$, $\cos \alpha$, and $\tan \alpha$).

Doubtless in the customary instruction in freshman mathematics too little attention has been paid to the functions $y = ax^n$, $y = a \sin mx$, and $y = k \cdot a^x$, and to the elementary applications of these functions and of the conic sections. Possibly in the future some way will be found to include in the freshman course, while preserving a logical treatment of the mathematical material, some applications which will be practical from the standpoint of the freshman. The present text does not appear to be successful either in logical treatment or in the presentation of practical material adapted to first-year students.

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An Introduction to the Theory of Automorphic Functions. By LESTER R. FORD, M.A. (Harv.) G. Bell and Sons, Limited, London, 1915. viii+96 pp. Price 3s. 6d.

THIS is No. 6 of the Edinburgh Mathematical Tracts and has its origin in a series of lectures on automorphic functions given by Mr. Ford to the Mathematical Research Class of the University of Edinburgh during the spring term of 1915.

Mr. Ford has endeavored to bring out “the concepts and theorems on which the theory is formed, and to describe in less detail certain of its important developments.” The tract is therefore conceived in the nature of an orientation rather than that of a treatise, and contains six chapters: I, Linear transformations; II, Groups of linear transforma-

tions; III, Automorphic functions; IV, The Riemannian-Schwarz triangle function; V, Non-euclidean geometry; VI, Uniformization.

In the chapter on linear transformations the proof, on pages 5-6, that the linear transformation is the most general one yielding a one-to-one correspondence between a z - and a z' -plane is incomplete. The most general function of this kind may be written in the form

$$z' = f(z) = \frac{A_m}{(z-g)^m} + \frac{A_{m-1}}{(z-g)^{m-1}} + \cdots + \frac{A_1}{z-g} + \phi(z),$$

where g is the only pole which may occur and where $\phi(z)$ is holomorphic in the entire z -plane. For, if ϕ were not holomorphic throughout (including $z = \infty$), $f(z)$ would have more than one place where $z' = \infty$, which is against the supposition of a one-to-one correspondence. $\phi(z)$ must therefore be a constant. The rest of the proof then follows as shown on page 6.

In the discussion of the regular solids, Chapter IV, the labeling of the regions in the stereographic projection of the octahedral group by the corresponding substitutions $1, S, T$, and their products would greatly benefit the student. It should also be shown, by one example at least, how polyhedral functions based upon these groups may be constructed.

The connection between non-euclidean geometry and groups of linear substitutions may be established more explicitly by Poincaré's own method. Another model of a very clear, brief, and effective demonstration of this proposition may be found in "Die Idee der Riemannschen Fläche," by H. Weyl, pages 148-159.

It seems to me that the exceedingly important subject of "uniformization," even in a mathematical tract, should have received a fuller treatment. Reference to the elementary example of the uniformization of curves of deficiency 1 by elliptic functions would have added interest to this chapter.

The bibliography of automorphic functions at the end is a most valuable feature of the little book. We miss a reference to Gauss. See remarks by Fricke in Gauss's Collected Works, volume 8, pages 102, 103, volume 3, page 477.

But I suppose that desiderata of all kinds regarding an introduction to the vast subject of automorphic functions, limited to 96 pages, vary from person to person.

It must be said, however, that within this space Mr. Ford has succeeded well in the task which he has set for himself.

ARNOLD EMCH.

A Course in Interpolation and Numerical Integration for the Mathematical Laboratory. By DAVID GIBB. (Edinburgh Mathematical Tracts, No. 2.) London, G. Bell and Sons, 1915. viii+90 pp.

A Course in Fourier's Analysis and Periodogram Analysis for the Mathematical Laboratory. By G. A. CARSE and G. SHEARER. (Edinburgh Mathematical Tracts, No. 4.) London, G. Bell and Sons, 1915. viii+66 pp.

THESE two little volumes of the series edited by Professor Whittaker treat some of the more essential parts of the subjects of interpolation and numerical approximation, the first being devoted chiefly to the non-periodic case of polynomial interpolation, the second mainly to trigonometric interpolation in the representation of periodic functions. In the first volume, after a very brief introductory chapter on finite differences, Chapter II is devoted to the various standard interpolatory formulas of Lagrange, Newton, Stirling, etc., and closes with a brief account of numerical differentiation. Chapter III, on the construction and use of mathematical tables, is in part devoted to explaining in detail the application of the foregoing principles to direct and inverse interpolation, and in part to special methods for computing tables of logarithms. Finally Chapter IV is concerned with numerical integration.

The second volume begins with a chapter which gives in barest outline and quite without proofs the main facts which the practical man must know about Fourier's series. This chapter closes with Bessel's elegant deduction of a finite trigonometric sum which gives the best approximate representation of a function in the sense of the method of least squares when the values of the function at equally spaced points are known. It is the evaluation of the coefficients of these finite sums (not of Fourier's series) which is considered in Chapter II by various methods, chief among which are the systematized methods of computation devised by Runge in the cases of 12 and 24 ordinates. Certain graphical methods are also explained, but the instruments which effect this interpolation mechanically are explicitly excluded. Chapter III, which is entitled Periodogram Analysis, is devoted to a discussion of