results obtained involve Fermat's quotient and Bernoulli's numbers.

19. In the *Quarterly Journal,* volume 45 (1914), pages 1–51, Professor J. W. L. Glaisher has calculated the first 27 eulerian numbers from certain recurring formulas and has shown that the method was especially advantageous when "curtate" formulas were employed. Mr. Joffe has verified Professor Glaisher's results and has extended the calculations to five more eulerian numbers by a different method based upon the formula

$$E_n = \sum_{m=0}^{n} (-1)^{m+n} e_{m,n},$$

where $e_{m,n}$ denotes $(1/2^m)\delta^{2m}0^{2n}$, and the quantities $\delta^{2m}0^{2n}$ are "central differences of zero." The successive terms $e_{m,n}$ are computed by a continuous process from the recurring formula

$$e_{m,n} = m[me_{m,n-1} + (m + \overline{m-1})e_{m-1,n-1}],$$

and the final values of E_n are verified by the formula

$$E_n = \sum_{m=1}^{n-1} (-1)^{m+n+1}[(m+1)(m+2) - 1]e_{m,n-1}.$$

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ON A CONFIGURATION ON CERTAIN SURFACES.

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The surfaces here under consideration are rational and are generated by conics. They may be represented birationally on the plane in such a way that, to the plane or hyperplane sections of a given surface of the given type, correspond curves of order n having in common an $(n-2)$-fold point P_0 and Δ simple points $P_1, P_2, \cdots, P_\Delta$. We suppose further that $\Delta = 2k$, so that the surface is of even order, that $n > 3$ and that $k > 2$. For simplicity, we suppose that the fundamental points $P_0, P_1, P_2, \cdots, P_{2k}$ are in generic position.

The generating conics on the surface are determined by the lines in the parametric plane through P_0. The $2k$ conics which correspond to the lines joining P_0 to $P_1, P_2, \cdots, P_{2k}$ are composite in such a way that the points of one component correspond to the points of the line P_0P_i, those of the other component to the directions through P_i.* We denote the former component right lines by the symbols $1, 2, \cdots, (2k)$, the latter by the symbols $1', 2', \cdots, (2k')$, respectively, so that the lines 1 and 1', 2 and 2', etc., constitute a composite generating conic.

The directrix curves of lowest order on the surface are 2^{2k-1} curves C^{n-2} of order $n-2$. These curves are determined as follows:

one, by the directions through P_0,

${}_{2k}C_2$ by right lines through two simple fundamental points,

${}_{2k}C_4$ by conics through P_0 and four simple fundamental points,

. . . .

one, by the curve of order k, which has a $(k-1)$-fold point at P_0 and passes through $P_1, P_2, \cdots, P_{2k}$,

wherein ${}_iC_j$ denotes the number of combinations of i things j at a time.

The C^{n-2} which is determined by directions through P_0 intersects the lines $1, 2, \cdots, (2k)$. We shall denote it by the composite symbol $1\ 2\ 3 \cdots (2k)$. If, in this symbol, we put, in all possible ways, primes on an even number of the component symbols we determine, for each of the remaining C^{n-2}, a symbol which indicates the lines on the surface that are intersected by it. Two such C^{n-2} which have 2σ component symbols unlike intersect in $\sigma - 1$ points as may be seen by counting the intersections, not at fundamental points, of the corresponding curves in the parametric plane. In particular, each C^{n-2} is intersected in $k-1$ points by a single C^{n-2} of the system.

We can choose sets of precisely $2k$ of the curves C^{n-2} in such a way that no two curves of the set intersect. In fact, if we birationally transform the parametric plane so that a given curve C_1^{n-2} corresponds to the directions at P_0, then the C^{n-2} which do not intersect C_1^{n-2} correspond to right lines through pairs of simple fundamental points. The C^{n-2} which correspond to the right lines joining a given simple fundamental point P_i to the remaining simple fundamental points form, with C_1^{n-2}, a set of the required type. Conversely,

* Clebsch, *Math. Ann.*, vol. 1, p. 266.

any given set can be represented in this way. Any $2k-1$ of the C^{n-2} of such a set determine a unique right line on the surface which intersects them all. Of the $2k$ lines determined in this way by a set, an odd number have primed symbols. Conversely, any set of $2k$ right lines on the surface, such that an odd number have primed symbols, determines a unique set of $2k$ C^{n-2} which do not intersect. The number of such sets is thus 2^{2k-1}. A given C^{n-2} belongs to $2k$ sets. Two given skew C^{n-2} both belong to two sets. Three skew C^{n-2} define a unique set.

Let S denote a set of the given type. Each curve of the set determines a unique C^{n-2} which intersects it in $k-1$ points and which is seen at once from its symbol to intersect each of the other curves of S in $k-2$ points. The $2k$ C^{n-2} determined in this way by the C^{n-2} of S are skew to each other. Hence they form a set S' of the given type. It follows at once from the method of formation that the correspondence between S and S' is involutorial.

We have supposed $n > 3$, $k > 2$. In case $k = 2$, there exist, in addition to the above sets of skew curves, eight sets of four skew C^{n-2} such that all the C^{n-2} of a set intersect a fixed line on the surface. Each line on the surface determines a set and the sets can be arranged in conjugate pairs.

In case $n = 3$, the C^{n-2} are right lines and the above configurations are components, merely, of known configurations of right lines on rational surfaces which have their plane sections of genus unity.

A similar theory can be developed when the number of simple fundamental points is odd, $\Delta = 2k + 1$, so that the surface is of odd order, but in this case the correspondence between the sets, as defined above, is not one to one.

URBANA, ILL.,
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