SHORTER NOTICES.

Robert of Chester's Latin Translation of the Algebra of Al-Khowarizmi, with an introduction, critical notes, and an English version. By LOUIS CHARLES KARPINSKI. University of Michigan Studies, Humanistic Series, Vol. XI. New York, The Macmillan Company, 1915. viii + 164 pp. Price $2.

OF the various kinds of contributions to the history of mathematics, those which are based upon a study of original sources are, of course, the most important. It was through contributions of this type that Boncompagni's *Bullettino* exercised its great influence; it is through these studies that the Abhandlungen zur Geschichte der Mathematik have proved so valuable; and it is through its articles based upon original sources that the *Bibliotheca Mathematica* has made its reputation. In this critical study of original sources, however, our own scholars have thus far been very backward. This is entirely natural, because it is first necessary to study the secondary sources in any new branch, and it takes time to discover a problem and to find the opportunity for assisting in its solution. On this account those to whom the story of mathematics has a charm have thus far, in our country, been chiefly occupied in learning the literature and in delving into the pages of Cantor and his predecessors, or in studying as time allowed such works as those of Heath or Braunmühl.

It is for this reason that the appearance of this work by Professor Karpinski is particularly noteworthy. Many scholars publish the results of their studies in one line or another from time to time, and these results are often noteworthy; but to few is it given, as it has been in this case, to suggest a new excursion into one of the by-paths of the academic grove of a country. And yet this is what has been done in the work under review, for it stands as the first noteworthy original study of a European version of an early classic in mathematics.

It is well known that Al-Khowarizmi, early in the ninth century, wrote a work bearing the title "algebr w'almuqabala." It is also well known that this work was translated into Latin by Robertus Retinensis (Ketenensis, de Ketene, Ostiensis, Astensis, or Cestrensis), commonly known today as Robert of Chester, one of the group of English scholars that revealed the most important scientific works of the Arabs to western Europe in the twelfth century. Here, however, the story comes to an end for most students. Occasionally some reader, browsing in the appendix to Libri's Histoire, comes across the Latin version ascribed to Gherardo of Cremona, and the Rosen translation (1831) from an Arabic manuscript in the Bodleian Library is accessible in any large scientific collection; but Robert of Chester's version has thus far remained beyond the reach of most students. Curiously enough, however, the work was at one time prepared for the printer, by Johann Scheybl, professor of mathematics at Tübingen, probably soon after 1550. For some reason Scheybl's text was never published, and this manuscript was not preserved, like various others of this writer, in the library of his university. After various wanderings it appeared in the stock of a German book dealer about fifteen years ago, and the writer of this review purchased it for a small sum, as an anonymous manuscript on mathematics, for the library of Columbia University. About a year later, upon examining it with some care, the name of Scheybl suggested that it might be one of his manuscripts, and photographs were made of other manuscripts in his hand at Tübingen. A comparison of the handwriting showed at once, and without question, that the Columbia manuscript was a lost one of Scheybl's. Moreover, it was seen that it contained a Latin version of Al-Khowarizmi which differed in various details from the one published by Libri and from the Arabic copy used by Rosen. It is this manuscript which Professor Karpinski has transcribed, translated, and annotated.

The form of the publication is very satisfactory. On the left-hand page is the Latin text; on the right-hand page is the translation; at the foot of each page are such notes as are necessary to show the variation of the Scheybl version from the Dresden and Vienna codices and from the Arabic manuscript used by Rosen. Professor Karpinski has wisely refrained from attempting a literal translation, since the transcribed text furnishes all needed material for the study of exact expressions; but he has given his readers that free type of translation which permits of a work being easily read and easily understood. All difficulties of any moment are removed by the extensive array of footnotes, and altogether there is little which one could desire that has not been given.

In one sense the Robert of Chester version is not as satisfactory as the one attributed to Gherardo of Cremona, but the latter had already been published by Libri, and hence it was desirable that the former should also be made available for study. In this connection it is also proper to mention another manuscript of Al-Khowarizmi's algebra, for a knowledge of which many students are indebted to Professor Karpinski. This version lay unnoticed in an Italian manuscript of the fifteenth century until the present reviewer happened upon it in the library of the well-known bibliophile George A. Plimpton, of New York, some years ago. This may possibly be the version of William of Luna, and some scholar should do for it what Professor Karpinski has done for the Robert of Chester translation.

Of the work of Al-Khowarizmi itself, this is not the place to speak, since we are concerned with the translation rather than the original. One problem concerning it may, however, be mentioned, namely, that which relates to the source of Al-Khowarizmi's knowledge. Essentially, the treatment is Greek, but no direct connection exists between it and such classical works as those of Euclid and Diophantus. Neither the problems nor the identical methods can be traced to any other source, although Al-Khowarizmi knew something of Hindu mathematics and the Greek authors were already becoming known in Bagdad where he was at work. The problem, therefore, is to determine whether any of this material is to be found in the works of minor Greek or Hindu writers, or possibly in the works of Persian and Chinese authors whose treatises have still to be critically examined.

The work closes with a carefully selected Latin glossary which will be helpful to students of mathematics of the medieval and renaissance periods. It is probably too much to expect that in any university series of this kind the reader is to be assisted by an index. Whether this is because of tradition or because of the economy of our universities it is hard to say; but we may be sure that on this occasion it is not due to the wishes of a student like Professor Karpinski. No one who, like the writer, has had occasion to refer to this work several times, and wishes to find such an item as Gherardo's supposed translation, can fail to regret the omission of this feature.

On the whole, it may be said that the work under review is a very noteworthy contribution to the study of sources in the history of mathematics.

DAVID EUGENE SMITH.

A First School Calculus. By R. WYKE BAYLISS. London, Longmans, Green and Company, 1915. xii + 288 pp.

THE pedagogical method used in this book is distinctly different from any found in the usual elementary calculus text. The author, a mathematical master at a boys' preparatory school in England, aims to teach the calculus to the youths by means of the question and answer method. Simple and definite questions on concrete problems concerning matter supposedly familiar to the youthful students are used to develop and fix the fundamental principles of the calculus. There are 180 pages of questions and suggestions; the answers to these cover 100 pages.

An equivalent of a meager high school course in mathematics seems sufficient as a prerequisite. Much of the work could be done orally; a private student might make considerable headway by using the text. Graphical work is minimized and included almost entirely among the answers.

No attempt is made to introduce rigor in the derivation of formulas. For example, the formulas based on the exponential function are developed from a practical consideration of the rate of increase of a sum of money placed at compound interest (continuous)—a concept with which all the students are supposed to be familiar. Or they are advised to draw a figure and use this to derive a formula. Or tables of trigonometric functions may be used to get average rates of increase and thus lead to general formulas. All of which, thoroughly rough and ready, seems like substituting a butcher's cleaver with a fairly dull edge for the scalpel in a surgical operation.

In the integral calculus much time and labor is saved by the following definition of integral: "We have seen that the symbol $D^{-1}f(x)$ denotes the expression for the *amount* of a quantity when its rate of increase is denoted by $f(x)$. The *amount* $D^{-1}f(x)$ is called the **integral** of the function $f(x)$." After which formulas may be applied in large chunks. And there is an everlasting amount of formal differentiating and integrating to be done.

The evaluation of the definite integral is arrived at through