

the summation process common to most calculus texts. Much smoother sailing is evident when areas, volumes, centroids and moments of inertia are found.

Needless to say, the law of the mean, extended law of the mean, various forms of remainders and their like are decidedly not included in the chapter on the expansion in series, nor are the various degrees of convergence considered. As the author puts it “. . . there are ‘pinnacles’ and ‘caverns’ which only the experienced mathematician should explore.”

The concluding chapter is headed The Borderland of Discovery. In other texts this is labeled Approximate Integration.

There is throughout the book much material of a rough and ready nature, which is well worth while and of great service in illustrating the fundamental principles of the calculus; there is certainly lacking the close reasoning which only the use of the method of limits can assure the calculus. The question and answer method might work well with a very limited number of students, though they would certainly have to be English because all units used are intensely British; but, with all the answers given in detail, we doubt very much if even the most conscientious students might not too often be tempted to “look in the book and see.”

ERNEST W. PONZER.

Mathematische Abhandlungen, Hermann Amandus Schwarz zu seinem fünfzigjährigen Doktorjubiläum am 6. August 1914 gewidmet von Freunden und Schülern. Berlin, Springer, 1914. Portrait, viii+451 pp.

THIS volume forms an imposing testimonial to the influence on the development of modern mathematics of the research and teaching of Schwarz. Of the thirty-four papers contributed by his friends and former students, a majority deal with subjects and methods brought out by him. It is impossible, within the limited space of this review, to do full justice to the rich contents of this volume, so that the reviewer must confine himself to mentioning a few of the papers which have been of particular interest to him, while regretting the necessity of passing in silence many noteworthy contributions.

C. Carathéodory gives a simplification of his former proof in the *Mathematische Annalen*, volume 72, of the most general existence theorem in conformal representation, and establishes

a number of cases in which the continuous one-to-one correspondence of the boundary points may be shown by Schwarz's principle of reflexion. L. Fejér proves some simple and elegant theorems on the convergence of power series, defining a conformal representation, on their convergence circle, and O. Hölder deals with the question of the variation of the solution of a differential equation when the form of the latter is varied. A. Hurwitz solves the problem, proposed by Weierstrass, of the possibility of defining the elliptic sigma function by its addition theorem, while P. Koebe has an article, also inspired by Weierstrass, on analytic functions possessing an algebraic addition theorem. E. Landau carries his researches on prime numbers into definite quadratic forms and pure cubic number fields, Ch. Müntz gives an elegant extension of Weierstrass's theorem on approximation by ordinary polynomials to such as involve non-integral powers, and E. Schmidt presents simple proofs of the fundamental properties of the Newtonian potential, beautiful by their unity of method. J. Schur investigates the expansion of a function in a series of characteristic functions of a positive definite kernel, M. Simon contributes an attractive sketch of the life and works of Sophie Germain, O. Toeplitz gives an example throwing much light upon the scope of Mercer's theorem in integral equations, and in the final paper of the volume, D. Hilbert deals with a general question in the theory of invariants, closely connected with his work in the early nineties.

T. H. GRONWALL.

Graphische Methoden. Von C. RUNGE. Leipzig, Teubner, 1915. iv+142 pp.

THIS book, which appears as No. 18 of Jahnke's collection of mathematical and physical texts, is a translation of the lectures delivered by the author in 1909-10 at Columbia University and published in 1912 as No. 4 of the publications of the Ernest Kempton Adams Fund for Physical Research.

Chapter I gives the means for performing graphically the four elementary operations on real numbers, the graphical calculation of polynomials in one variable and of linear functions of n variables, including the solution of a system of linear equations, and ends with the representation of complex numbers in the Gaussian plane.

Chapter II deals with graphs of functions of one variable,