of the Gibbs-Wilson notation is admitted, the note is misleading. For had brevity been the chief aim of my paper the notation of Burali-Forti and Marcolongo could have been made to compare very favorably with Professor Wilson's compact reproduction of the formulas in the Gibbs notation. (Compare the analytic statement of the theorems in the two notations.) However, as one of the purposes of my paper was to exhibit the operational feature of the system of Burali-Forti and Marcolongo, obviously the Gibbs-Wilson notation did not lend itself to this end. Moreover, since the notation of Burali-Forti and Marcolongo is not so well known in this country, some explanation seemed to be necessary.

Professor Wilson says in closing: "The use of words like grad, div, rot is hampering: we no longer write Cubus $\overline{m}$ Census $\bar{p}$ 16 rebus aequatur 40 for $x^3 - 8x^2 - 16 = 40$." Everybody admits the last part of this statement. But we still use for particular kinds of functions or operators such symbols as log, sin, cos, etc., arcsin, etc., sinh, cosh, etc. That the use of such "words" as grad, div, rot is hampering, seems to be a matter of opinion, since they may be used interchangeably with other symbols in both notations.

It is unfortunate that Professor Wilson introduced cartesian coordinates into his proof, since a coordinate system has no proper place in vector analysis. But this seems to be characteristic of the Gibbs-Wilson system. In fact Burali-Forti and Marcolongo have pointed out how the dyadics of Gibbs constantly depend on cartesian coordinates,* a nonlinear system.

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SHORTER NOTICES.

Grundlehren der Mathematik. Der zweite Band des ersten Teils: *Algebra.* By EUGEN NETTO. Leipzig, Teubner, 1915. xii+232 pp.

THE Grundlehren der Mathematik, für Studierende und Lehrer is a series of four volumes on the elements of mathematics appearing from the press of B. G. Teubner under joint authorship as follows: Part I (two volumes), Die Grundlehren

* Burali-Forti et Marcolongo, Transformations linéaires, 1912, p. 147.

der Arithmetik und Algebra, by E. Netto and C. Färber; Part II (two volumes), Die Grundlehren der Geometrie, by W. Fr. Meyer and H. Thieme. One of the objects of this series is to present the fundamentals of mathematics with particular regard to the influences which a century's advances in the science, especially in the direction of greater accuracy and completeness, have had upon its elementary phases. In addition it is proposed to present in the second volume of each part extensions which will serve as an introduction to enquiries which a deeper understanding of elementary mathematics renders feasible. There is, therefore, an interest attached to the selection of topics which the authors have made, and the volume Algebra, by Dr. Netto, is in this way noteworthy. We find in this algebra nothing about convergency of series. A topic affording choice between two methods of treatment, one involving analytic functions, as for instance the theory of the roots of unity, is consistently developed by the algebraical method. Certain arithmetical topics usually found in algebras, such as permutations and combinations, are also omitted. On the other hand, there is given a rather complete treatment of differentiation of rational functions, including topics in maxima and minima, two proofs of the fundamental theorem of algebra, a chapter on cyclotomy, and a proof of the Abel-Ruffini theorem.

The developments in the text follow an elementary mode, and contain numerous particular examples solved out. In the first chapter determinants are treated as summations, use being made, in determination of the signs of the terms, of -1 to the power $[\alpha_1\alpha_2 \cdots \alpha_n]$, where the latter symbol stands for the number of transpositions from the order $1, 2, \cdots, n$ afforded by the permutation $\alpha_1, \alpha_2, \cdots, \alpha_n$. With the aid of the properties of this symbol the standard theorems are developed as theorems on summations. The second chapter, on rational functions, discusses and illustrates definitions, graphical representations, continuity and limits, and derivatives are introduced by means of the expansion of a polynomial $f(x+h)$ in powers of h. Among the examples of maxima and minima of rational functions occurs that of the function $f(x) = (a-x)^\alpha (b-x)^\beta$, $(a < b)$, where α, β are parameters representing only positive integers > 1. The special value of such a problem is that it separates into a definite number of cases (four), according to the possibilities (α even, β even), (α even, β odd), etc.

Such a problem, requiring separation into cases, affords to the student a real illustration of scientific method.

Chapter three on integral functions takes up constructions by Lagrange's method of interpolation, Euclid's algorithm, partial fractions, and reducibility. A trial method of resolving a polynomial with integral coefficients into factors with integral coefficients is next discussed and illustrated. There follow Eisenstein's and Gauss's theorems on this problem, the former finding application in Chapter ten in a proof of the irreducibility of $(x^p - 1)/(x - 1)$ (p a prime number). There is then given a discussion of a domain of rationality and functions in a domain. After a brief discussion in Chapter four of elementary properties of equations, Chapter five gives the theory of linear dependence and the practice of solution, both approximate and exact, of systems of linear equations. The next chapter, on resultants and discriminants, contains a proof that the dialytic eliminant is the resultant, a treatment of subresultants and common roots of two equations, and the construction of discriminants in determinant form and in terms of the roots. The seventh chapter gives the solution, in much completeness, of the general equations of the second, third, and fourth orders, following in the latter cases the method ot Euler.

The proofs given in Chapter eight that every algebraical equation has a root are respectively the first proof of Gauss, given in his Helmstädt dissertation in 1799, and a proof published by Cauchy in 1821.

Symmetric and alternating functions are treated in Chapter nine, the roots of unity by number-theoretic considerations in Chapter ten. The eleventh contains the theory of cyclotomy, regular polygons, and the f-nomial periods. Cyclic equations and their solution by radicals, and reciprocal equations form the subjects of the twelfth chapter. The succeeding chapters then lead up to the proof of the Abel-Ruffini theorem, treating substitution groups and their functions, and the solvability of algebraic equations in general. There then follows a chapter on transformation, with definitions and illustrations of invariants and covariants and reduction of the general n-ary quadratic form to a sum of squares, and a final chapter on Sturm's theorem.

Netto's treatment is, as a whole, particularly commendable, containing many a deft touch marking the handiwork of a master of his craft.

The standard of typography is high. A few misprints may be noted however: On page 3, line 7, αn should read α_n; on page 30, line 6, for $y \pm 2kn$ read $y \pm 2k\pi$; on page 59, last line, read a_k for a_κ; on page 85 in line 19, $a_1 d - a_1 d$ should read $ad_1 - a_1 d$. The proof of page 106 has not been well read. There occur three notations for the same function on this page, viz., $g(x)$, $f'(x)$, and $f_2(x)$. The order of f is m at the top of the page, is changed to n at the middle and so used in two determinants, there being no comment on the change, and the order m is restored at the bottom of the page. In line 3 from the bottom $(1/n^{n-1})R(f_2, f_1)$ should be $(1/n^{n-2})R(f_2, f_1)$. Moreover it is not good usage, we believe, to begin a sentence with a mathematical symbol instead of a capitalized word, as is done in the theorem given at the top of this page. In line 11 of page 115 the last β in the line is wrong font. On page 120 in line 15, $2\sqrt{(a_1^2 - a_0a_1)/a_0^2}$ should be $2\sqrt{(a_1^2 - a_0a_2)/a_0^2}$. In line 3 from the bottom of page 122 read a_4 for a_6. In line 2 of page 123 read x_3x_4 for x_3x_6. The numbering of the formulas in the region of page 123 is confused. Equation (27) referred to in line 6 of this page does not occur in the chapter. This renders line 16 on page 125 unintelligible although it may be a misprint of "Nun liefert (21) wegen (23)." On page 213 in line 4 read y_2 for y_2'.

In the way of general criticism the reviewer thinks it might be urged that the treatment of invariants in the book is much too brief. Quite probably this subject is to be expounded at greater length in the volumes on geometry. But if it could have been found feasible to introduce the notions of invariancy in connection with the solutions of the equations of orders less than 5, at sufficient length to show, for instance, that the roots of the resolvent cubic of the quartic equation are irrational invariants of index 2, the rôle of invariants in the elements of algebra would have been rendered more evident.

O. E. GLENN.

Solid Geometry. By WILLIAM BETZ, A.M., and HARRISON E. WEBB, A.B. With the editorial cooperation of PERCY F. SMITH. Ginn and Company, 1916. xxii+177 pp. Price $0.75.

ON account of the existence of so many other interesting and important topics in mathematics which can be offered to the