$\zeta(\theta)$ and we then have the result that if

$$r^2 = [x - \xi(\theta)]^2 + [y - \eta(\theta)]^2 + [z - \zeta(\theta)]^2,$$

$$\alpha = t - \frac{r}{c}, \quad \beta = \frac{x - \xi(\theta) - i[y - \eta(\theta)]}{r + [z - \zeta(\theta)]},$$

the definite integral

$$V = \int_{\theta_1}^{\theta_2} F(\alpha, \beta, \theta) \frac{d\theta}{r}$$

satisfies the wave equation, provided $\theta_1$ and $\theta_2$ are roots of the equations

$$G_1(\alpha, \beta, \theta) = 0 \quad \text{and} \quad G_2(\alpha, \beta, \theta) = 0$$

respectively. The case in which $F$, $G_1$ and $G_2$ are independent of $\beta$ is of special interest. $V$ may then be regarded as the velocity potential of a chain of sources of sound each of which is only active for a certain interval of time which may be different for different sources.

THROOP COLLEGE OF TECHNOLOGY,
PASADENA,
*October* 6, 1917.

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## LITERATURE OF PURE MATHEMATICS.

*Historical Introduction to Mathematical Literature.* By G. A. MILLER. New York, Macmillan, 1916. 14 + 302 pp. Price $1.60.

IT is difficult to overestimate the extent of inspiration which may emanate from interesting exposition of problems and wonders of science in a form intelligible to those who are not deeply versed in the subject. How much richer must be the intellectual outlook of thousands throughout the world who have read: Science and Hypothesis, The Value of Science, and Science and Method! Is it hard to believe that the future historian may some day tell us that the very notable stage of advancement of astronomy in America in 1910 was not a little due to Simon Newcomb's remarkable gifts of popularization of his subject, exercised through written and spoken word in magazine, book, society, and congress during

a long series of years—educating and inspiring the people to a frame of mind which called for organization, equipment, and cooperation greater than had been?

For many years Professor Miller has been one of a small group of Americans who have very frequently written and spoken with authority in the interests of the mathematical amateur and inquiring general student. There can be little doubt of the success of their efforts to arouse in America more widespread enthusiasm and deeper appreciation of the values involved in the study of mathematics, through discussion of some of its historical, philosophical, geometrical, and analytical aspects. In this connection apart from expository articles (more particularly in the study of groups) Professor Miller has published many articles and addresses attractive not so much on account of their stimulating style, for in this they are sometimes lacking, but rather on account of their timely and highly informational content, very clearly, suggestively, and thoughtfully expressed. We welcome then this unique work,* with similar characteristics, in which much that had been published is coordinated, developed, and expanded, and combined with new features.

We are told that the volume is based on a course of lectures which were designed to supplement the regular work of mathematical courses, and that in it the author (1) "aimed to meet the needs of a textbook for synoptic and inspirational courses which can be followed successfully by those who have not had extensive mathematical training"; and (2) provided a work which "may also be used as a textbook for a first course in the history of mathematics, especially by those teachers who believe with its author that such a first course should largely concern itself with recent mathematical events and developments." The author states that in his presentation he attempted to guide the mathematical student to "points from which he can overlook domains of considerable extent in order that he may be able to form a somewhat independent judgment as regards the regions which he might like to examine more closely." Let us now consider the work in some detail.

The "General Observations" (35 pages) of the first chapter

* G. Loria's Guida allo Studio della Storia delle Matematiche (Milano, 1916), which was published almost simultaneously with this work has very different ideals. See the review by D. E. Smith in this BULLETIN, 1916, vol. 23, pp. 136–139.

are made under the headings: Changes during the nineteenth century; history of mathematics; developed parts of the history of mathematics; periods in the history of mathematics; usefulness of the history of mathematics; first decade and a half of the twentieth century; American mathematics. Since details of purely domestic interest are recorded in connection with the references to the American Mathematical Society, it is a wonder that one fact of international interest is not mentioned, namely, that since 1913 the society has enjoyed "the possibly unique distinction of holding almost simultaneous meetings in different cities."*

The second chapter on "Mathematical Literature in General" is somewhat longer. In the first section on "Types of recent mathematical literature" the discussion is mainly confined to English, French and German periodicals. Except for slips in the dates of the *Cambridge Mathematical Journal* and of the *Cambridge and Dublin Mathematical Journal*, the statements concerning English periodicals are accurate; but surely wrong inferences may be drawn from the two pages devoted to *The Ladies' Diary* (1704–1840) and the *Gentleman's Diary or the Mathematical Repository* (1741–1840).† We are told that the latter "may be regarded as a scientific mathematical journal and it contains a number of important problems and solutions relating to elementary mathematics." And then: "These journals [the diaries] did much to keep alive in England a general interest in mathematics, and they contain a number of theorems relating to elementary mathematics which were rediscovered in later times." Since, for the period they cover, no other English mathematical journal is mentioned, one natural inference from the above would be that there was no other of equal importance. And yet, for example in the eighteenth century‡ there were two journals devoted *wholly*§ to mathematics and certainly of greater scientific significance. The one, *Miscellanea Curiosa Mathematica*,‖ contained William Chapple's notable paper (1746) with its results concerning the distance between the centers of the

* In this BULLETIN, January, 1914, vol. 20, p. 170.

† No reference is made to the amalgamation of the almanacs into *The Lady's and Gentleman's Diary* (1841–1871).

‡ Similar illustrations could be given concerning the nineteenty century.

§ In the latter part of the century not more than a third of the *Diaries* was of a mathematical nature.

‖ Fourteen numbers published in London, 1745–1754.

inscribed and circumscribed circles of a triangle, and other observations anticipating formulas of Euler* and discussions by Poncelet, Steiner, and Jacobi. The other journal, *The Mathematical Repository*,† contains the first enunciation (March, 1799) of the well-known theorem concerning Wallace's line.‡

On page 44 reference might be made to De Morgan's sketch of the "Mathematical Society of Spitalfields" in his Budget of Paradoxes.

Prince Boncompagni's *Bullettino di bibliografia e di storia delle scienze matematiche e fisiche* is listed in a section on "special historical periodicals," and it is stated that a "peculiarity of this periodical is that very extended references are given even when they relate to matters where the literature s easily accessible, and it is probable that these extensive references have tended to decrease the use of the journal, since they are often so numerous as to burden the reader." A much less trivial "peculiarity," and a source of real exasperation to the historical inquirer, is due to the fact that there are probably few (perhaps not even two) sets of the *Bullettino* which are exactly alike throughout. As the periodical was set up and printed in his palace Boncompagni frequently stopped the press after some numbers were printed in order to introduce additions and changes. In this way it is said that several different editions of the same number may be found.§

In the section on "Encyclopedias and other works of reference" a misleading statement has been made in connection with "the Encyclopédie Méthodique Mathématique, in three volumes, by d'Alembert and others, 1784–89." My edition is in at least four volumes, the last of which, Recueil de Planches du Dictionnaire Encyclopedique des Mathématiques, was published in M.DCC.XCVII. I have written "at least" because there are three more volumes published with the following titles: (1) Dictionnaire des Jeux, faisant suite au

* *Novi Commentarii . . . Academiae . . . Petropolitanae*, tome 11 (1765) 1767, pp. 103–123.

† London, 1795–1835.

‡ Proposition I, in a paper by William Wallace entitled "Mathematical Lucubrations," vol. II, p. 111. It is implied in a parabola theorem which he published in the fourth number of the *Repository* (October, 1797).

§ Cf. *Isis*, June, 1914, tome II, pp. 133–134. This reference is to an article by G. Sarton containing an annotated list of "Soixante-deux revues et collections relatives à l'histoire des sciences," to which attention might naturally be drawn in such a section as that which we are considering.

Tome III des Mathématiques, 1792; (2) Dictionnaire des Jeux familiers ou des Amusemens de Société, faisant suite au Dictionnaire des Jeux, annexé au tome III des Mathématiques; An V [1797]; (3) Dictionnaire des Jeux mathématiques contenant l'Analyse, les Recherches, les Calculs, les Probabilités et les Tables numériques, publiés par plusieurs célèbres Mathématiciens, relativement aux Jeux de Hasard et de Combinaisons, et suite du Dictionnaire des Jeux, an VII [1799].

In this same section we find also: "The most useful work of reference for the purpose of finding the periodical literature of the nineteenth century is the Subject Index of the Royal Society of London Catalogue of Scientific Papers, volume 1, 1908. . . . Since the beginning of the twentieth century this work is supplemented annually by volume A of the International Catalogue of Scientific Literature." And why not give reference to volume 2, 1909 (Mechanics), to volume 3, 1912–14 (Physics, but with much of value for the mathematician), to volumes B (Mechanics), E (Astronomy), J (Geography, which includes geodesy) and C (Physics)? The fact of the matter is that our author disregards applied mathematics practically altogether, so that a more accurate title of his book would be "Historical Introduction to the Literature of Pure Mathematics."

In another section of the second chapter, on "Mathematical tables," a reference to the work edited by Horsburgh* would have usefully supplemented that to the 1873 Report of the British Association for the Advancement of Science. The following statement is perhaps a trifle misleading: "In 1911 H. Andoyer of the University of Paris published a table giving the logarithms of the trigonometric functions . . . under the title Nouvelles Tables Trigonométriques Fondamentales, as it is expected to become fundamental for other tables." The volume in question was the first of three volumes and it consists mainly of a set of logarithmic tables for sines, cosines, tangents and cotangents. The second and third† volumes (1915–16) are principally occupied with tables of natural trigonometric functions. It seems strange to find no

* Modern Instruments and Methods of Calculation. A Handbook of the Napier Tercentenary Exhibition (1914).

† This volume may have appeared after Professor Miller's book was out.

mention in this section of the outstanding works by Crelle,* Peters,† and Bauschinger.‡

Other topics of the chapter are: Mathematical societies, international mathematical congresses, periodicals on periodicals and books on books, and collected works.

The third chapter, entitled "General Historical Questions Relevant to Mathematics," includes discussion of such subjects as: Definitions of the term mathematics, divisions and subdivisions of mathematics, some dominant concepts, notation and terminology, errors in mathematical literature, and living mathematicians. As the 1908 and 1916 editions of the Index du Répertoire Bibliographique des Sciences Mathématiques have been published by Delsman in Amsterdam, a change is desirable in a statement on page 89. Another instance, among several, of national rather than international outlook occurs on page 123 where, by indirection, H. H. Stephenson's Who's Who in Science (International) which passed through several editions (1912–1914) is condemned as a source of information as to "mathematical standing," while American Men of Science is characterized as a "much more useful type of work."

In the next three chapters fundamental developments in arithmetic, geometry and algebra are discussed. Arithmetical topics are: Existence of an infinite number of prime numbers with Euclid's proof, sieve of Eratosthenes, irrational quantities and numbers, fundamental operations of arithmetic, systems of notation, and Fermat's last theorem. Is it not misleading to give, without comment, 1228 as the date of the "noted Liber Abaci by Leonardo of Pisa"?

The headings for the chapter in geometry are: The Pythagorean theorem, area of the circle, area and volume of the sphere, regular geometric forms, the triangle. It is recalled that the French and the Germans frequently refer to the

* A. Crelle, Calculating tables giving the products of every two numbers from one to one thousand and their application to the multiplication and division of all numbers above one thousand. New edition by O. Seeliger, 1908.

† J. Peters: 1. New calculating tables for multiplication and division, by all numbers of from one to four places. 1909. 2. Siebenstellige Logarithmentafel der trigonometrischen Funktionen, für jede Bogensekunde des Quadranten. 1911.

‡ J. Bauschinger and J. Peters, Logarithmic-trigonometrical tables with eight decimal places containing the logarithms of all numbers from 1 to 200,000 and the logarithms of the trigonometric functions for every sexagesimal second of the quadrant. 2 vols. 1910–11.

Pythagorean proposition as "the asses' bridge." Other names might have been mentioned: "Theorem of the married woman" by the Greeks; "Chaise of the little married woman" by the Hindus (Bhascara); and "figure of the bride" by the Persians.* The first edition of J. J. I. Hoffman's collection of proofs of the proposition is referred to; a "zweyte verbesserte und mit einigen neuen Beweisen vermehrte Ausgabe" was published in 1821.

On pages 175–6 it is stated that Gauss proved (Disquisitiones Arithmeticæ, section 7) that "a necessary and sufficient condition that a regular polygon of a prime number of sides can be inscribed in a circle by means of ruler and compass is that this prime be of the form $2^m + 1$." If the reference to the Disquisitiones had been omitted no criticism could have been made of this statement; but as it is, incorrect inferences are likely to be drawn.†

A reference is given to Casey's Sequel to Eculid, 1888, for information concerning the geometry of the triangle. It may be noted that it is this particular edition alone which contains the supplementary chapters on this subject.

Some of the topics in the chapter on Developments in Algebra are: Fundamental theorem of algebra, determinants, numerical equations, domains of rationality, invariants and binomial theorem.

The seventh and last chapter contains brief sketches of twenty-five prominent deceased mathematicians: Euclid, Archimedes, Apollonius of Perga, Diophantus of Alexandria, Viète, Descartes, Fermat, Newton, Leibniz, Euler, Lagrange, Gauss, Cauchy, Steiner, Abel, Hamilton, Galois, Sylvester, Weierstrass, Cayley, Kronecker, Hermite, Cremona, Lie, and Poincaré. In the sketch of Euler it is stated that "he arrived, in 1748, at the celebrated formula

$$e^{ix} = \cos x + i \sin x."$$

$i$ for $\sqrt{-1}$ appears to have been first used by Euler in 1777. In 1722 Roger Cotes gave the equivalent, in words, of $\sqrt{-1}\varphi = \log(\cos\varphi + i \sin\varphi)$. In the sketch of Gauss reference is made to his proof of the fundamental theorem of

* Cf. E. Lucas, Récréations mathématiques, tome II, 1883, or 1896, p. 130; E. Fourrey, Curiosités géométriques, Paris, 1907, p. 64.

† In more than one place in Klein's Famous Problems of Geometry misstatements occur in this connection.

algebra, to various parts of the Disquisitiones Arithmeticæ, to his work on the theory of orbits and theory of errors, and to his geometric theory of complex numbers. His fundamental monograph in the theory of surfaces is not mentioned, even though the best edition of it was published by Princeton University. It is not accurate to refer to even the major portion of Euclid's book On Divisions (of figures) as "still extant" (page 219). And finally, in the sketches of Hermite and Poincaré our author does not seem to be sufficiently clear as to the use of some French terms: (1) Hermite "became professor also at the École Polytechnique and at the Faculté des Sciences" (page 261); for "Faculté des Sciences" substitute "Université de Paris"; (2) Poincaré "was elected a member of l'Académie des Sciences" (page 269), and "was received as member of l'Académie Française"* (page 271), and "received two prizes from the Paris Academy" (page 272); after each of the first two expressions quoted add "de l'Institut de France," although there is no ambiguity in the second case; for "Paris Academy" (which may be interpreted as having an entirely different meaning from the one intended†) substitute "Institut de France" or "Académie des Sciences de l'Institut de France." It is surely of interest to bring out more clearly the fact that Poincaré was a member of these two Académies of the five composing the Institut de France—the greatest honor which France has in her gift for the élite of her scientists. I am under the impression that J. L. F. Bertrand was the only other scientist who has been similarly honored.

An appendix of 15 pages contains an annotated list of bibliographies and encyclopedias, histories, and books on teaching and philosophy of mathematics. The reviewer sees no advantage in giving partial indications of the first, instead of the last, editions of the works mentioned; and even in this respect there is a slip in connection with Cantor's Vorlesungen. The current edition of the Encyklopädie der Elementar-Mathematik by Weber and Wellstein contains four, not three, volumes (page 280).‡

* This Académie has a "président" but not a "directeur" (p. 271).

† *Cf.* this BULLETIN, March, 1910, vol. 16, p. 329.

‡ There are fairly obvious typographical errors on page 87, line 13; page 130, line 2 from bottom; page 289, line 17; and page 293, line 5. The spelling of the name Čebysëv on page 133, line 5, does not agree with that later on the page or in the index; similarly Lobačevskỳ on pages

The Author Index and Subject Index occupy about ten pages.

To sum up: Professor Miller has written a valuable, original, and very entertaining book, containing much out-of-the-way information difficult of access elsewhere. It deals only with pure mathematics, and in this field emphasizes considerably the subjects of groups, theory of numbers, and theory of equations. While the treatment is often, perhaps necessarily, scrappy (by reason of its "synoptic" nature) it has been shown above that the separate sections do not always treat the subject with the breadth to be expected in a volume designed to guide a student to "points from which he can overlook domains of considerable extent." Either the amateur or the professional mathematician must find much of interest within its covers, however, and the work is heartily recommended.

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## SHORTER NOTICES.

*A History of Elementary Mathematics with Hints on Methods of Teaching.* By FLORIAN CAJORI. New York, The Macmillan Company, 1917. viii + 324 pp. Price $1.75.

Two of the most unsatisfactory literary labors that members of our guild are called upon to undertake are first, the publication of a new edition of an old work, using plates of many years' standing; and second, the review of an edition prepared under such circumstances. Probably the most severe critic of the work in hand is Professor Cajori himself, and if he had been free to do so he would undoubtedly have revised the work more radically than most of his readers would or could undertake to do. The book was written more than twenty years ago, and the world knows more about the subject than it did at that time; it has found better ways of overcoming certain difficulties in the matter of presentation of material; and it has the problem of a history of mathematics better in hand.

125 and 272 (add to index) differs from the index form; for Graup, page 161, line 4 from bottom, read Graap; for McMahon, page 293, read MacMahon.

Here and above the slips or errors are practically all of somewhat minor importance. As a whole Professor Miller's work is exceedingly accurate.