3 sextactic points are contacts of tangents from the flexes \( P_3 \). The 6 contacts of tangents from the sextactic points are the points \( P_{12} \). The 12 contacts of tangents from \( P_{12} \) in turn are the points \( P_{24} \), and so on ad infinitum.

University of Oregon.

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RELATED INVARIANTS OF TWO RATIONAL SEXTICS.

BY PROFESSOR J. E. ROWE.

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Let the parametric equations of the \( R_6^6 \), the rational curve of order six in three dimensions, be

\[
x_i = \delta_i t^6 + a_i t^5 + 6b_i t^4 + 15c_i t^3 + 20d_i t^2 + 15e_i t + 6f_i + g_i \quad (i = 1, 2, 3, 4),
\]

and let the parametric equations of the \( R_6^6 \), the rational plane curve of order six, be of the form

\[
\begin{align*}
x_1 &= \alpha t^6 + a + bt + ct^2 + dt^3 + et^4 + ft^5 + gt^6, \\
x_2 &= \beta t^6 + a' + bt + ct^2 + dt^3 + et^4 + ft^5 + gt^6, \\
x_3 &= \gamma t^6 + a'' + b''t + c't^2 + d't^3 + e't^4 + f't^5 + g't^6.
\end{align*}
\]

It is well known that all plane sections of the \( R_6^6 \) are apolar to a doubly infinite system of binary sextics, and that all line sections of the \( R_6^6 \) are apolar to a triply infinite system of binary sextics. We shall let the four binary sextics \( \delta_i t^6 \) of (1) be four linearly independent sextics of the apolar system of the \( R_6^6 \), and the \( \alpha_i t^6, \beta_i t^6, \gamma_i t^6 \) of (2) be three linearly independent sextics of the apolar system of the \( R_6^6 \). Our purpose is to point out briefly the relation between the invariants of the \( R_6^6 \) and the invariants* of the \( R_3^6 \).

By means of the twelve equations

* This relation must not be confused with the correspondence between invariants of the \( R_6^a \) and covariant surfaces of the \( R_6^a \).
\[ a_ia - b_ib + c_ic - d_id + e_ie - f_if + g_ig = 0, \]
\[ (3) \]
\[ a_ia' - b_ib' + c_ic' - d_id' + e.ie' - f_if' + g_ig' = 0, \]
\[ a_ia'' - b_ib'' + c_ic'' - d_id'' + e.ie'' - f_if'' + g_ig'' = 0 \]

\[(i = 1, 2, 3, 4),\]

it may be easily proved that the four-rowed determinants of the matrix of the coefficients of \( \delta_ii^6 \) of the type \( |abed| \) are proportional to the complementary three-rowed determinants of the matrix of the coefficients of \( \alpha_i\delta_6, \beta_i\delta_6, \gamma_i\delta_6 \) of the type \( |ef'g'| \).

Let \( T \) denote the substitution of the three-rowed determinants of (2) for the proportional four-rowed determinants of (1), and \( T^{-1} \) the inverse substitution.

Invariants of the \( R_2^6 \) are combinants of the four sextics \( \delta_ii^6 \), and conversely, and these are rationally expressible in terms of the determinants of the type \( |abcd| \). Invariants of the \( R_2^6 \) are combinants of \( \alpha_i\delta_6, \beta_i\delta_6, \gamma_i\delta_6 \), and conversely, and these are rationally expressible in terms of the determinants of the type \( |ab'e''| \). The combinants of \( \delta_ii^6 \) are implicit invariants of the \( R_2^6 \) which become explicit invariants of the \( R_2^6 \) after the application of \( T \). Similarly, combinants of \( \alpha_i\delta_6, \beta_i\delta_6, \gamma_i\delta_6 \) are implicit invariants of the \( R_3^6 \) which are transformed into explicit invariants of the \( R_3^6 \) by means of \( T^{-1} \). Hence any explicit invariant \( I \) of the \( R_3^6 \) is transformed into an explicit invariant \( I' \) of the \( R_3^6 \) by means of \( T \). Similarly, \( T^{-1} I' = I \). It is evident that the order of \( I \) in the \( |abcd| \) is the same as that of \( I' \) in the \( |ab'e''| \). We shall now mention a few illustrations of this relation.

If \( U' \) is the undulation invariant of the \( R_2^6 \), \( T^{-1} U' = U \) is the stationary line invariant of the \( R_3^6 \). From \( P \), the pantatactic plane invariant of the \( R_3^6 \), we obtain \( TP = P' \), the cusp invariant of the \( R_3^6 \). Similarly, from \( Q \), the quinqueseant line invariant of the \( R_3^6 \), we derive \( TQ = Q' \) whose vanishing defines an \( R_3^6 \) such that any six of its collinear points have parameters apolar to a binary quintic. If \( N = 0 \) is the necessary and sufficient condition that the \( R_3^6 \) have a node, \( TN = N' = 0 \) defines an \( R_3^6 \) which has one secant that cuts out a cyclotomic set of parameters.

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