3 sextactic points are contacts of tangents from the flexes $P_3$. The 6 contacts of tangents from the sextactic points are the points $P_{12}$. The 12 contacts of tangents from $P_{12}$ in turn are the points $P_{24}$, and so on ad infinitum.

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RELATED INVARIANTS OF TWO RATIONAL SEXTICS.

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Let the parametric equations of the $R_3^6$, the rational curve of order six in three dimensions, be

\[ x_i = \delta^6_{ij} = a_i t^6 + 6b_i t^5 + 15c_i t^4 + 20d_i t^3 + 15e_i t^2 + 6f_i t + g_i \quad (i = 1, 2, 3, 4), \]

and let the parametric equations of the $R_2^6$, the rational plane curve of order six, be of the form

\[ x_1 = \alpha^6 t = a + bt + ct^2 + dt^3 + et^4 + ft^5 + gt^6, \]
\[ x_2 = \beta^6 t = a' + bt' + ct'^2 + dt'^3 + et'^4 + ft'^5 + gt'^6, \]
\[ x_3 = \gamma^6 t = a'' + bt'' + ct''^2 + dt''^3 + et''^4 + ft''^5 + gt''^6. \]

It is well known that all plane sections of the $R_2^6$ are apolar to a doubly infinite system of binary sextics, and that all line sections of the $R_3^6$ are apolar to a triply infinite system of binary sextics. We shall let the four binary sextics $\delta^6_{ij}$ of (1) be four linearly independent sextics of the apolar system of the $R_3^6$, and the $\alpha^6, \beta^6, \gamma^6$ of (2) be three linearly independent sextics of the apolar system of the $R_2^6$. Our purpose is to point out briefly the relation between the invariants of the $R_2^6$ and the invariants* of the $R_3^6$.

By means of the twelve equations

* This relation must not be confused with the correspondence between invariants of the $R_2^n$ and covariant surfaces of the $R_3^n$. 
\[a_1a - b_1b + c_1c - d_1d + e_1e - f_1f + g_1g = 0,\]
\[a_2a' - b_2b' + c_2c' - d_2d' + e_2e' - f_2f' + g_2g' = 0,\]
\[a_3a'' - b_3b'' + c_3c'' - d_3d'' + e_3e'' - f_3f'' + g_3g'' = 0\]
\((i = 1, 2, 3, 4),\)

it may be easily proved that the four-rowed determinants of the matrix of the coefficients of \(\delta_i^6\) of the type \(|abcd|\) are proportional to the complementary three-rowed determinants of the matrix of the coefficients of \(\alpha^6, \beta^6, \gamma^6\) of the type \(|ef'g'|\).

Let \(T\) denote the substitution of the three-rowed determinants of (2) for the proportional four-rowed determinants of (1), and \(T^{-1}\) the inverse substitution.

Invariants of the \(R_2^6\) are combinants of the four sextics \(\delta_i^6\), and conversely, and these are rationally expressible in terms of the determinants of the type \(|abcd|\). Invariants of the \(R_3^6\) are combinants of \(\alpha^6, \beta^6, \gamma^6\), and conversely, and these are rationally expressible in terms of the determinants of the type \(|ab'c'|\). The combinants of \(\delta_i^6\) are implicit invariants of the \(R_2^6\) which become explicit invariants of the \(R_3^6\) after the application of \(T\). Similarly, combinants of \(\alpha^6, \beta^6, \gamma^6\) are implicit invariants of the \(R_3^6\) which are transformed into explicit invariants of the \(R_2^6\) by means of \(T^{-1}\). Hence any explicit invariant \(I\) of the \(R_3^6\) is transformed into an explicit invariant \(I'\) of the \(R_2^6\) by means of \(T\). Similarly, \(T^{-1} I' = I\). It is evident that the order of \(I\) in the \(|abcd|\) is the same as that of \(I'\) in the \(|ab'c'|\). We shall now mention a few illustrations of this relation.

If \(U'\) is the undulation invariant of the \(R_2^6\), \(T^{-1} U' = U\) is the stationary line invariant of the \(R_3^6\). From \(P\), the pantatactic plane invariant of the \(R_3^6\), we obtain \(TP = P'\), the cusp invariant of the \(R_2^6\). Similarly, from \(Q\), the quinqueseant line invariant of the \(R_3^6\), we derive \(TQ = Q'\) whose vanishing defines an \(R_2^6\) such that any six of its collinear points have parameters apolar to a binary quintic. If \(N = 0\) is the necessary and sufficient condition that the \(R_3^6\) have a node, \(TN = N' = 0\) defines an \(R_2^6\) which has one secant that cuts out a cyclotomic set of parameters.

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