subarc AA_0 of AB, and D is B or (if $A_0 \neq B$) any point of the subarc A_0B of AB, then the subarc CD of AB contains at least one point of $[S]$.

This theorem is tacitly assumed by Lennes in his proof of Theorem 7 (§ 4).

All the above mentioned theorems thus hold if the term simple continuous arc is defined without the use of the word "bounded." Using this definition of an arc, I now prove the

THEOREM. *A simple continuous arc is bounded.*

Proof. Suppose AB is an arc which is not bounded. Let $[S_1]$ consist of A and all points S_1 of AB such that the arc AS_1 is bounded. Let $[S_2]$ consist of all other points of AB. By hypothesis both $[S_1]$ and $[S_2]$ exist. No point of S_2 is between A and a point of $[S_1]$. Since AB is connected, $[S_1]$ contains a limit point P_1 of $[S_2]$ or $[S_2]$ contains a limit point P_2 of $[S_1]$. In the first case any triangle t_1 containing P_1 contains an arc a_1 of AB containing P_1. The arc a_1 contains a point Q_2 of $[S_2]$. The arc AP_1 of AB is contained in a polygon p_1. Therefore the subarc $AQ_2 = AP_1 + P_1Q_2$ lies entirely within a polygon (Lennes, Theorem 15, § 2), and is bounded, contrary to hypothesis. In the second case any triangle t_2 about P_2 contains an arc a_2 containing P_2, a_2 contains a point Q_1 of $[S_1]$, AQ_1 is contained in a polygon, and therefore $AP_2 = AQ_1 + Q_1P_2$ is contained in a polygon and is bounded, contrary to hypothesis. Thus in either case the supposition that AB is not bounded leads to a contradiction.

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THE TRANSFORMATION OF A REGULAR GROUP INTO ITS CONJOINT.

BY DR. J. E. MCATEE,

(Read before the American Mathematical Society October 26, 1918.)

1. CONSIDER a regular substitution group G of order g. All the substitutions on the same letters that are commutative with every substitution of G form a group G', of order g, called the conjoint of G. These groups are conjugate.* If G is abelian, $G = G'$. In the contrary case the statement that a

* Finite Groups, Miller, Blichfeldt and Dickson, 1916, p. 35.

substitution t of order 2 exists that transforms G into G' occurs in the literature.* Our purpose is to exhibit such a t.

Let the substitutions of G be arranged as follows:

$$G = 1, s_2, s_3, \cdots, s_e, s_k, \cdots, s_g,$$

where $1, s_2, s_3, \cdots, s_e$ is the cyclic group generated by s_2 $(s_i = s_2^{i-1}, i = 2, \cdots, e)$. Consider the two square arrays

$$\begin{array}{llllllll}
1, & s_2, & s_3, & \cdots, & s_e, & s_k, & \cdots, & s_g \\
s_2, & s_2^2, & s_3s_2, & \cdots, & s_es_2, & s_ks_2, & \cdots, & s_gs_2 \\
s_3, & s_2s_3, & s_3^2, & \cdots, & s_es_3, & s_ks_3, & \cdots, & s_gs_3 \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
s_e, & s_2s_e, & s_3s_e, & \cdots, & s_e^2, & s_ks_e, & \cdots, & s_gs_e \\
s_k, & s_2s_k, & s_3s_k, & \cdots, & s_es_k, & s_k^2, & \cdots, & s_gs_k \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
s_g, & s_2s_g, & s_3s_g, & \cdots, & s_es_g, & s_ks_g, & \cdots, & s_g^2
\end{array}$$

and

$$\begin{array}{llllllll}
1, & s_2, & s_3, & \cdots, & s_e, & s_k, & \cdots, & s_g \\
s_2, & s_2^2, & s_2s_3, & \cdots, & s_2s_e, & s_2s_k, & \cdots, & s_2s_g \\
s_3, & s_3s_2, & s_3^2, & \cdots, & s_3s_e, & s_3s_k, & \cdots, & s_3s_g \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
s_e, & s_es_2, & s_es_3, & \cdots, & s_e^2, & s_es_k, & \cdots, & s_es_g \\
s_k, & s_ks_2, & s_ks_3, & \cdots, & s_ks_e, & s_k^2, & \cdots, & s_ks_g \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
s_g, & s_gs_2, & s_gs_3, & \cdots, & s_gs_e, & s_gs_k, & \cdots, & s_g^2
\end{array}$$

Each row of these squares is a permutation of the elements of the first row. Each of these permutations represents a substitution and we may assume, without loss of generality, that the substitutions thus obtained from the first square are those of G. For simplicity in what follows we shall call these substitutions $1, u_2, u_3, \cdots, u_g$. The substitutions $1, u_2', u_3', \cdots, u_g'$ represented by the second square are those of G'.†

The substitution represented by the row Gs_2 in the first

* Ibid., p. 46.

† Loc. cit.

square is

$$u_2 = s_1 s_2 s_3 \cdots s_e \cdot s_k s_h s_v \cdots s_i s_m s_n \cdots s_x s_q \cdots,$$

where $s_1 = 1$. The substitution represented by the row $s_2 G$ in the second square is

$$u_2' = s_1 s_2 s_3 \cdots s_e \cdot s_k s_i s_x \cdots s_h s_m s_q \cdots s_v s_n.$$

To show that the replacements indicated in u_2 and u_2' are correct we note first that the first cycles in u_2 and u_2' are the same, since s_2 is commutative with each of its powers. Also, since $s_h = s_k s_2$ and $s_i = s_2 s_k$, we have $s_2 s_h = s_i s_2$. Therefore s_i is replaced in u_2 by what s_h is in u_2'. Moreover, if s_h is replaced by s_v in u_2, s_i by s_x in u_2', s_x by s_q in u_2 and s_v by s_n in u_2', it results that s_m is replaced by s_n in u_2 and by s_q in u_2'. In fact, the product $u_2 u_2'$ gives s_h replaced by s_n, while in $u_2' u_2$ we have s_h replaced by the product $s_m s_2$. Hence $s_m s_2 = s_n$. Similarly, we have that s_m is replaced by s_q in u_2'. From this we see that the substitution

$$t = (s_h s_i)(s_v s_x)(s_n s_q)$$

transforms u_2 into u_2' so far as the elements involved are concerned. Continuing this, we ultimately obtain a substitution t of order 2 that transforms any power of u_2 into the same power of u_2'. Consider next the substitutions u_j and u_j' represented by the rows $G s_j$ and $s_j G$ respectively (s_j not in the cyclic subgroup generated by s_2). In u_j we have s_2 replaced by what s_j is in u_2' and in u_j' we have s_2 replaced by what s_j is in u_2. Now G and G' are simply isomorphic and this isomorphism may be established such that all the substitutions begin with s_2. Then t transforms G into a group that is simply isomorphic with G' and such that the first two letters in corresponding substitutions are the same. Since G and G' are regular it follows that t transforms G into G'.

2. In the substitution u_2 of G we have the cycle

$$s_k s_k s_2 s_k s_2^2 s_k s_2^3 \cdots s_k s_2^{e-1},$$

where $s_k s_2^i$ is to be considered as one letter. Let $s_p = s_k s_2 s_k^{-1}$. Then, since $s_p^i = s_k s_2^i s_k^{-1}$, we have $s_p^i s_k = s_k s_2^i$, that is, in the substitution of G' corresponding to the row $s_p G$ will be found the above cycle. This shows that the substitutions of

G and G' are composed of the same cycles combined differently when G is not abelian. Take for example,

$$G = 1,\ (abc)(def),\ (acb)(dfe),\ (ad)(bf)(ce),\ (ae)(bd)(cf),\ (af)(be)(cd),$$

$$G' = 1,\ (abc)(dfe),\ (acb)(def),\ (ad)(be)(cf),\ (ae)(bf)(cd),\ (af)(bd)(ce).$$

URBANA, ILLINOIS,
September 24, 1918.

CORRECTIONS.

PROFESSOR G. LORIA has kindly pointed out the fact that the curves discussed in the first part of my article "Some Algebraic Curves" published in volume 25, pages 85–87 of the BULLETIN are special cases of curves discussed in his treatise "Spezielle Algebraische und Transcendente Ebene Kurven," volume I, pages 390–4 (1910). However the main theorem of the section, viz., the rth polar of B with respect to C_n is C_{n-r} is not found in Loria's treatise.

J. H. WEAVER.

On page 472 of the BULLETIN for July, 1918, line 10, for certain functions t read certain functions of t; line 4 from bottom, for $l^{2i\pi/p}$ read $e^{2i\pi/p}$.

On page 53 of the BULLETIN for November, 1918, line 11 from bottom, for field read fluid. On page 56, line 4, for tanh (μu) read tanh $(\frac{1}{2}\mu u)$.

NOTES.

THE total membership of the American Mathematical Society on January 1, 1919, was 723, including 79 life members. The total attendance of members at all meetings held in 1918, including sectional meetings, was 222; the number of papers read was 137. The number of members attending at least one meeting was 155. Accessions to the Library in 1918 included 74 periodicals and 12 non-periodicals, making a total