

## SCRITTI MATEMATICI.

*Scritti matematici offerti ad Enrico D'Ovidio in occasione del suo LXXV genetliaco, 11 Agosto 1918, e pubblicati per cura di FRANCESCO GERBALDI e GINO LORIA.* Torino, Fratelli Bocca, editori, 1918. 8vo, 386 pages. Lire 30.

THE title of this volume sufficiently indicates its general character. A beautiful portrait forms the frontispiece, representing Professor D'Ovidio as he will be remembered by many hundreds of Italians as well as by a number of Americans who, like the writer, have had the good fortune, at one time or another, to be among those in attendance at the University of Turin.

There is a short preface written by Professors Gerbaldi and Loria in which is set forth the *raison d'être* for the volume. To quote the opening sentence—"On the approach of the day on which an inflexible law would retire Senator Enrico D'Ovidio from the university chair, there arose in the minds of many students whom he has had in his long and glorious career as a teacher, the pleasant idea of choosing this occasion—which coincides with his 75th birthday—to manifest to him their sentiments of unalterable affection and, at the same time, to present to him their sincere good wishes *ad multos annos*."

Of the 103 persons contributing to the expense of publication, and whose names appear directly following the preface, 47 have been actual students under Professor D'Ovidio, and 20 have been, at one time or another, directly associated with him as *Assistenti* in the University of Turin. The list contains many names well known to students of mathematics the world over, and bears witness to the great influence Professor D'Ovidio has had upon the growth of mathematics and upon the teaching of mathematics, not alone in Italy, but, mainly perhaps through his disciples, throughout the civilized world. This influence has been exerted from the University of Turin for more than forty years.

Following the list of contributors is a catalogue of 91 publications by Professor D'Ovidio. Some of these are problems or solutions of problems; two or three are elementary texts; a number are commemorative addresses or biographical

papers; the majority deal with problems in geometry; several, the most important perhaps, are concerned with the theory of binary forms. A glance at the titles of these papers leads to the conviction that Professor D'Ovidio's great influence has come about rather more through personal contact as a teacher than through published writings, and goes to show that there is such a thing as creative teaching as well as there is creative scholarship—a fact worth noting in a time when so much emphasis is placed upon the latter function and apparently so little is thought about the former.

It would be out of place here to review in detail the twenty-one papers forming the body of the volume. The titles and authors of these papers with perhaps a word or two indicating the content must suffice.

I. "Su alcune classi particolari di sistemi continui di quadriche, e sui rispettivi involuppi." CORRADO SEGRE, Turin.

The lines of ordinary 3-space can be imaged as points upon a  $V_4^2$  in  $S_5$ . A study of point loci upon the  $V_4^2$  leads to properties of ruled surfaces in  $S_3$ . In particular, sections of the  $V_4^2$  by the planes in  $S_5$  are images of quadratic reguli in  $S_3$ . Professor Segre is thus led to what he calls concatenated quadric surfaces and quartic curves of the first kind, and to systems of concatenated quadrics and quartics. A quadric surface and a quartic curve of the first kind lying on the surface are concatenated, or chained together, if a simple quadrilateral inscribed in the latter has its sides lying upon the former. The present paper is evidently an outgrowth of a recent paper by Professor Segre in *Annali di Matematica* (3) 27, (1918), page 151.

II. "Le frazione continue di Halphen." FRANCESCO GERBALDI, Pavia.

The continued fractions of Halphen (*Traité des Fonctions elliptiques*, second volume) arise from the expression  $\frac{\sqrt{X} - \sqrt{Y}}{x - y}$ , where  $X$  and  $Y$  are the values of a biquadratic expression when the variable assumes the values  $x$  and  $y$ , respectively. Halphen, and Jacobi before him, made use of elliptic functions in arriving at the development of irrational expressions like the above in continued fractions. Professor Gerbaldi returns to the more elementary methods of Abel in the present paper.

Two other papers on the same subject have appeared;—one in Volume 53 of *Atti di Torino*, and the other in Volume 51 of *Rendiconti R. I. Lombardo*.

III. “Le cubiche gobbe aventi ciascuno all’infinito tre punti reali e distinti.” GINO LORIA, Genoa.

In 1879 D’Ovidio published an exhaustive study of twisted cubics making use of the symbolic notation of binary forms (*Memorie R. Accademia Torino* and *Giornale di Matematiche*). Professor Loria here studies the cubic hyperbolas, using only elementary means to arrive at the properties of these curves.

IV. “Intorno ad un tipo notevoli di sistemi lineari di reciprocità degeneri tra spazi ad  $n$  dimensioni.” EUGENIO TOGLIATTI, Turin.

The study of projective correspondences between hyperspaces leads to the consideration of types of degenerate transformations (cf. Bertini; *Introduzione alla Geometria proiettiva degli Iperspazi*). The present paper follows and is an extension of a note by the same author published in *Atti di Torino*, 1916–17.

V. “Sulle congruenze  $W$  di cui una falda focale e una quadrica.” ALESSANDRO TERRACINI, Turin.

One may consider a correspondence set up in which homologous points are the foci on the same ray of a congruence. If, in this correspondence, asymptotic lines on the focal mantels of the congruence are homologous lines, the congruence is called a  $W$ -congruence. The author here studies the  $W$ -congruences in which one of the focal mantels is a quadric.

VI. “Alcune osservazioni relative ai problemi secondarii della balistica esterna.” GUIDO FUBINI, Turin.

A study of the effect of the wind and of the rotation of the earth upon projectiles, with the hope of modifying the classical differential equations involved so that theoretical results will be in closer accord with experimental data.

VII. “Sulle curve che posseggono una infinità continua di corrispondenze algebriche.” GUIDO CASTELNUOVO, Rome.

A curve of genus greater than unity does not admit of an infinity of birational transformations into itself (Schwarz,

*Crelle*, 1875). Castelnuovo here studies those curves each of which is transformable into itself by a continuous infinity of non-birational algebraic correspondences and arrives at a generalization of the Schwarz theorem. The paper is based upon the classic memoir of Hurwitz (*Mathematische Annalen*, 1886).

VIII. "Le oscillazione armoniche nelle antenne radiotelegrafiche direttamente eccitate." LUIGI LOMBARDI, Naples.

This paper contains results of some experiments with radio apparatus at the polytechnic school in Naples.

IX. "Sugli integrali semplici di 1<sup>a</sup> specie appartenenti ad una superficie algebrica." FRANCESCO SEVERI, Padua.

A development of one of Severi's brief notes communicated to the Académie des Sciences, Paris, in 1911 (*Comptes Rendus*, volume 152, page 1079).

X. "Sopra alcune applicazioni della teoria dell'urto." EMILIO ALMANZI, Rome.

A brief study of the theory of impact in a system of moving bodies with application, in particular, to three spheres.

XI. "Generalizzazione di una trasformazione di d'Ocagne." ANGELO PENSA, Turin.

The transformations of d'Ocagne may be found explained in an article by M. d'Ocagne, *American Journal*, volume 11 (1889), pages 55-70. Pensa here generalizes the problems raised by d'Ocagne (solved in particular cases by himself and others) and reaches general results by means of vector analysis.

XII. "Estensione e studio di un metodo di sommazione generico di Borel." GUSTAVO SANNIA, Cagliari.

This paper follows one by the same author published in *Rendiconti del Circolo Matematico di Palermo*, volume 42, 1917. Both papers are based upon Borel's summation formula (*Leçons sur les Séries divergentes*, Gauthier-Villars, 1901).

XIII. "Sopra la propagazione di onde in un mezzo indefinito." ERNESTO LAURA, Pavia.

This paper deals with the motion of waves in a fluid medium when, in particular, the waves are reflected from a fixed surface. It follows an article by the same author published in *Atti di Torino*, 1915.

XIV. "Problemi sulla determinazione delle linee sghembe."

MATTEO BOTTASSO,\* Messina.

This paper deals with certain generalizations of the transformation of Combescure (Bianchi, *Lezione di Geometria Differenziale*, second edition, page 40). It follows a note by Bottasso published in *Atti di Torino*, 1917-18.

XV. "Riflessioni sopra alcuni principii della teoria degli aggregati e della funzioni." BEPPO LEVI, Parma.

A logico-mathematical discussion dealing in particular with the Zermelo postulate (*Mathematische Annalen*, volume 59).

XVI. "Nuovo metodo per la risoluzione diretta dell'equazione  $ax + by = c$  in numeri interi e positivi, quando i tre numeri noti  $a$ ,  $b$ ,  $c$  sono interi e positivi." GIUSEPPE BERNARDI, Bologna.

A short paper whose title sufficiently indicates its content.

XVII. "Un interessante problema di geodesia pratica."

NICODEMO JADANZA, Turin.

A discussion of the problem: To determine the mean error of two sides of a geodetic triangle when the third side and the three angles are measured.

XVIII. "Resto nelle formule di interpolazione." GIUSEPPE

PEANO, Turin.

A proof of an interesting theorem in determinants from which results, in particular, the well known Lagrange interpolation formula together with a remainder.

XIX. "Questioni elementari di massimo e minimo." FILIBERTO CASTELLANO, Turin.

Some problems in maxima and minima treated by algebraic equations and inequalities.

XX. "Sulle varietà algebriche a tre dimensioni a superficie-sezioni razionali." GINO FANO, Turin.

An abstract from a memoir with the same title published in *Annali di Matematica*, (3), volume 24 (1915).

XXI. "Introduzione alla teoria della forme in più serie di variabili." GIOVANNI GIAMBELLI, Messina.

A generalization of the theory of connexes (Clebsch-

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\* Died Oct. 3, 1918.

Lindemann). The paper follows two others on the same subject published in Turin by the author in 1910 and 1912.

One cannot turn aside from reading these papers, however cursorily, without a feeling of profound admiration for the scholars who have made the book possible, and especially at a time when the strain of the war was still in force. Each paper is a distinct contribution to knowledge or else a fuller development of such a contribution recently commenced. The total is a wholly worthy epitome of scientific activity even in normal times. It must, indeed, be a source of great satisfaction to Professor D'Ovidio to have so distinct a proof of the esteem with which his many students, associates, and friends regard his long service and his personal qualifications as an inspiring teacher. One cannot do better, in this connection, than to quote again from the preface. "And we are certain that to the loved teacher our publication will be doubly gratifying in as much as it serves also to show how Italy, in the tragic hours in which we live—not less than in the more grave and decisive periods of her earlier struggles for redemption—has not ceased to feed the sacred flame of science."

L. WAYLAND DOWLING.

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### SHORTER NOTICES.

*Plane Geometry.* By E. LONG and W. C. BRENKE. New York, The Century Company, 1916.

THIS text has several very good features. First, before a theorem is demonstrated a method of attack is given. Second, frequent use is made of algebra and thereby many blind proofs are avoided. Third, construction work is introduced early in the course. Fourth, areas are introduced before proportion. Fifth, a little trigonometry and analytics is given.

The main fault with the book is that it contains quite a number of inaccurate statements, e. g., "Place the triangles with their longest sides together," page 32; page 107,  $c^2 = h^2 + (b - a)^2$  is only true if  $A$  is acute and that is not at all necessary; page 204, the definitions of the trigonometric functions are incorrect.

F. M. MORGAN.