COLLEGE ALGEBRAS.

College Algebra. By E. B. Skinner. New York, The Macmillan Company, 1918. vi + 263 pp.

Advanced Algebra. By W. C. Brenke. New York, The Century Company, 1917. vii + 196 pp.

A First Course in Higher Algebra. By Helen A. Merrill and Clara E. Smith. New York, The Macmillan Company, 1917. xiv + 247 pp.

Recent textbooks written for the ordinary algebra course in college are apt to be scrutinized with other questions in mind beside the usual inquiries as to whether the traditional subject matter has been well presented and whether the problems have been so wisely selected and graded that they will be of actual assistance to the student in mastering the principles involved. For example: 1. Does the content or the manner of presentation show any modification due to the effects of those attacks upon mathematics as a required study which gave such a sharp challenge to the algebra of the high schools a few years ago? 2. Are the subjects which have been stressed in this text the topics which are usually selected for those "combination courses" in freshman mathematics which include algebra? 3. Has the desire of scientists to push back the elementary calculus into the earlier years of the college course had any effect upon this text?

In attempting to answer some of these questions regarding the three texts listed above, it has seemed advisable not to try to consider each separately, but rather to compare them. They will, of course, have much in common, but they may have enough differences to make such a comparison interesting. It is worth noting that the writers represent different types of institutions situated in different parts of the country. The authors of the first two books are professors in state universities (Wisconsin and Nebraska), while the authors of the third are in a New England college (Wellesley).

As the preface usually reveals the author's point of view, it may be well to quote here certain selected portions that seem to have especial bearing upon the questions in mind.

Professor Skinner says: "The shortening of the time given to algebra in the secondary school, together with the great extension of the elective system and the consequent placing of mathematics in competition with a score of new subjects, has made some modification of the traditional college algebra absolutely necessary. The first important change has been to put the college algebra upon a more elementary basis. . . . The second change is one which the writer believes will in the end prove to be of great advantage, not only to algebra, but to mathematics in general. The college teacher has been obliged to exert every effort to make his work of interest to his students. To this end the subject has been made more concrete, applications to the affairs of everyday life have been emphasized, processes have been made more direct, and the road to the mathematics of the sophomore year has been shortened. . . . I have tried to emphasize the immediately practical side of algebra by drawing freely upon geometry, physics, the theory of investment, and other branches of pure and applied science for illustrative examples. An amount of space greater than usual has been devoted to the study of the functions which occur most frequently in practical work. . . . I have not hesitated to omit a number of topics that are ordinarily included in textbooks on college algebra." . . .

Professor Brenke says: . . . "The first chapter and portions of the next three chapters are reviews of elementary algebra and are intended to make a close connection with the work of the high school. The preparation of the students in the average freshman class varies so greatly that such reviews are almost indispensable to establish a common basis for further progress. Graphic methods are introduced early and freely used. This is done both for the immediate utility of these methods and to serve as an introduction to Analytical Geometry. Numerous simple applications are contained in the exercises and problems, which will serve to establish some connection between theory and practice. Logarithms are introduced in Chapter III, so as to make this method of computation available, early in the course, for the numerical valuation of unknowns from given data." . . .

Professors Merrill and Smith say: "This book is an outgrowth of the conviction of the authors that Higher Algebra, to be worthy of the name, must employ advanced methods, and that the method which chiefly marks advanced work in analysis is that of limits. In all but a few chapters the work is based upon limits, the proofs being made as rigorous as seems advisable for immature students. . . . The ordinary Algebra course in college covers a semester's work—about forty-five class appointments. It has been found by actual use that in this time Chapters III, IV, V, VI, VII, XII can be covered, while Chapter X has been taught in connection with a course in Trigonometry." [The titles of these chapters are Determinants, Variables and their Limits, Differentiation of Algebraic Functions, Convergency of Series, Development of Functions in Series, Theory of Equations, and Logarithms (X).] "The chapters on Rational and Irrational Numbers [I and IX] are intended for reference rather than for detailed study, while the chapters on Permutations, Combinations and Probability [II], Partial Fractions [VIII], Complex Numbers [XI], and Integration [XIII] may be substituted for other chapters as subjects of study, or serve for reference in later work. No mathematical knowledge is presupposed beyond the usual course in Elementary Algebra, except that a knowledge of the meaning of the trigonometric tangent is assumed in § 105 [Chapter V] while in Chapter XI" . . . "The text has as its main ends to broaden the students' thought by introducing them as early as possible to some of the most beautiful and most fruitful methods in analysis, with a few of their results; to give an acquaintance with the simplest notions of the Calculus, . . .; and to open up to students some vision of the possibilities of later work along lines here treated only briefly. The students who are interested to do more than the assigned work have been kept constantly in mind."

In comparing these texts it may be wise to focus the attention upon three important subjects; logarithms, functions, and graphical representation, which would certainly be included in a "combination course" in freshman mathematics.

The space devoted to logarithms varies from twelve pages (Brenke) to twenty-six (Merrill and Smith). Professor Brenke adds a four-place table of logarithms of numbers and Professor Skinner has this and five other tables at the end of his book (powers and roots, important constants, compound interest and compound discount tables, and American experience table of mortality). The relative position of the first presentation of the subject is more significant than the number of pages. In the first book it is the fourth of eighteen chapters, being preceded by introduction, algebraic identities, and powers and roots. In the second text the third of thirteen chapters is on logarithms and the binomial theorem, the first two being on a review of the four fundamental operations and on theory of exponents and surds and imaginaries. Both these texts return to the subject of logarithms later. The position which the chapter on logarithms occupies in the third text has already been noted. In the first two texts, where the aim is to give the student an early introduction to the use of logarithms in numerical calculations, the first presentation contains the usual subject matter on common logarithms with sets of exercises including not only the purely arithmetic variety but also applied problems from geometry, physics, and compound interest. The first text adds two pages, with diagrams, on the logarithmic scale and the slide rule. In Chapter XIV (Progressions and Compound Interest) in the section on the compound interest law, the student is introduced to e. The last two sections of the last chapter (infinite series) are on the logarithmic series and series as a means of computation. In the second text natural logarithms are mentioned in a note to a problem at the end of Chapter VIII (infinite series) and in the next chapter (computations, approximations, differences and interpolation) there are sections on the computation of natural logarithms and common logarithms.

In the third text the chapter on logarithms opens with a long quotation from Napier's preface to his *Descriptio*, which will be much appreciated by students,—especially his reference to the "many slippery errors." It might be noted in passing that there is an appropriate quotation at the head of every chapter in this book. The treatment of logarithms in this text stands out in strong contrast to the method of the other two. After giving the definitions and the simple theorems, the authors say: "These theorems show that if the logarithms of all numbers to some base can be found, arithmetical processes can be greatly shortened." Then, after discussing the use of logarithms in solving exponential equations and adding historical notes on Napier and Briggs, they say, "The following sections give a method of computing the logarithms of numbers to any desired base." (It must be recalled that limits, derivative of an algebraic function, and series have been studied before this chapter.) Fifteen pages later the student is told 'The results obtained by these and other methods of computing logarithms are arranged in various forms, . . . modern tables are extremely easy to use after a little practice. As full instructions for using the tables are ordinarily included in each book, only a few general principles are given here." Two pages later there is a list of eleven exercises. This book does not contain a table of logarithms of numbers.

The place which graphic methods occupy in Professor Brenke's book is well stated in his preface. In Chapter IV (Linear Equations) after the algebraic solution of an equation in one unknown, the author discusses coordinates, the graph of the "expression" $2x - 1$ and the theorem "The graph of the equation $y = ax + b$ is a straight line." Throughout this chapter and the next, the author uses this plan of following the algebraic solution by the graphic. In Chapter V under "the graphic solution of two simultaneous equations in two variables, the one linear, the other quadratic," the student is given the standard equation of the circle, ellipse, parabola, and hyperbola and the equation of the rectangular hyperbola and the general equation of the second degree. He meets the terms asymptotes and conic section. It is not until the next chapter (Ratio, Proportion, Variation) that the term *function* is defined. According to the index, this is the only time that the term is used in the text. (In passing, it might be noted that the third book contains no index and that the first has a good one, and that the brief index in this text is quite lacking in one place at least, for it does not contain the word graph (or graphical), although the subject of graphs is so much stressed.)

In the first text the chapter that follows the one on logarithms is entitled "Functions of a Variable — Graphical Representation." On the first page, after a careful explanation of the term function, there is the statement, "The notion of a function is one of the most important ideas with which mathematics has to do." After giving many varied illustrations, the author explains coordinates and the graph of a function and then deals with the linear integral function (and its graph), uniform motion, rational integral quadratic function, maximum and minimum values of quadratic functions, the power function, variation, discontinuous functions, statistical graphs, interpolation of values of functions, zeros and infinities of a function and implicit functions. The greater part of the next chapter (Quadratic Equations with One Unknown) is devoted to the algebraic solution and the relation between roots and coefficients. But in the section on "geometric interpretation," its relation to the quadratic function of the last chapter is brought out. In Chapters VII, VIII, IX, and XI (systems of linear equations, and systems containing non-linear equations, inequalities, polynomials, and equations of any degree) the notion of a function is never lost sight of. It is to be noted that graphs are employed as needed in all these chapters, while in Chapter VIII the standard forms for the different species of conics are given.

In the third text the three and a half page introduction which is inserted between the preface and the first chapter gives information regarding the authors' attitude toward the place of graphical representation in their text. The introduction opens with the statement: "The following topics from elementary algebra are inserted for reference." [1. Graphical representation, (two pages, including the graph of $y = x^2 - 4x + 3$). 2. Roots of quadratic equations. 3. Binomial theorem. 4 and 5. Formulas for arithmetic and geometric progressions.] Assuming that graphical representation is a subject with which the student is already sufficiently familiar, it is employed in the text wherever it is deemed useful. This book resembles the first in giving a central position to the notion of the function. But the methods of presentation in the two books are quite different. According to the selection of material suggested in the preface, Professors Merrill and Smith begin with determinants and then, turning to variables and their limits, introduce the function on the second page of the new chapter and proceed to a discussion of algebraic functions, limits and infinitesimals (including theorems on limits), infinity, and continuity of functions. In the next chapter (twenty-six pages) the student is led up to the idea of the derivative and learns of theorems on derivatives and studies successive differentiation, geometric interpretation of the derivative, applications, and maximum and minimum values.

It is natural at this point to inquire whether either of the other texts goes into the question of derivatives. In the index of the second book the word derivative does not occur and the word limit appears but once,—the limit of a variable is defined in the chapter on infinite series. In the first text the word limit is used for the first time in connection with infinite geometric progressions (Chapter XIV). In Chapter XVII (Sequences and Limits), limits of variables are defined and discussed. This fourteen page chapter contains also explanations of the derivative of a power function, of the geometric interpretation of the derivative, and of maxima and minima.

A more detailed study of these texts will, of course, reveal other interesting material, and it will deepen the reader's conviction that the second book follows the traditional lines more closely and that, while the authors of the first and third texts differ radically from one another in their aims, each has succeeded in presenting an attractive book that has a distinct place.

E. B. COWLEY.

NOTES.

THE twenty-seventh summer meeting and ninth colloquium of the American Mathematical Society will be held at the University of Chicago, extending through the week September 6–11. The regular sessions of the Society and also those of the Mathematical Association of America will occupy the first days of the week, without conflict of hours. The joint dinner will be held on Tuesday evening. The Colloquium will open on Wednesday and extend to Saturday noon. It will consist of two courses of five lectures each by Professors G. D. BIRKHOFF, of Harvard University, and F. R. MOULTON, of the University of Chicago. Titles and principal topics of the lectures follow:

Professor BIRKHOFF: "Dynamical systems." The last forty years have witnessed fundamental advances in the theory of dynamical systems, achieved by Hill, Poincaré, Levi-Civita, Sundman, and others. The lectures will expound the general principles underlying these advances, and will point out their application to the problem of three bodies as well as their significance for general scientific thought. The following topics will be treated:

Physical, formal, and computational aspects of dynamical systems. Types of motions such as periodic and recurrent motions, and motions asymptotic to them. Interrelation of types of motion with particular reference to integrability and