mathematician may find the working out of a particular example to require in some cases a considerable amount of labor. Unless one has already completed a card catalogue or a note book containing his own solutions of such a wide range of examples as is to be found in Forsyth's Treatise, a work like the present will prove of great value.

The examples worked out in the German edition of the Treatise included only those contained in the first and second English editions. As is well known, the later editions contained a great many additional examples. The present volume includes the solution of these. All the examples have been worked out by Professor Forsyth himself, and, with possibly three exceptions, all were found to be solvable in the usual sense of the term.

CHARLES N. MOORE.

Space, Time and Gravitation; an Outline of the General Relativity Theory. By A. S. EDDINGTON. Cambridge, University Press, 1920. vii + 218 pp.

EDDINGTON has a pleasant style even when engaged in technical exposition. His Stellar Movements makes very interesting reading for any mathematician who likes to see what mathematics has recently done toward unraveling the structure of the sidereal universe. This style is a necessity when one tries to write a semi-popular account of Einstein's new theory. On the whole Eddington has succeeded in making the matter clear without appeal to too much mathematics. Of course the person who has absolutely no mathematical outlook beyond the high-school course will have difficulty in appreciating even the Prologue; but let us say a college graduate who has had his calculus, taught in no too formal fashion,—he will find the book possible. The physicist, the not too ignorant philosopher will welcome the chance to study the theory in its elemental simplicity. Einstein's own treatment and that of his followers is about as instructive to the beginner as lectures on generalized (Lagrangian) coordinates would be in collegiate physics as a first treatment of mechanics or Lamé's work as an introduction to the notion of potential.

Eddington begins with a Prologue on What is Geometry written in the form of a Platonic dialogue between an experimental physicist, a pure mathematician, and a relativist.

The three points of view are well done and spiritedly. He should have had a philosopher, but probably he shrank from abandoning his clear and definite style in an attempt to accord or even record such divergent points of view as are taken by Whitehead and Walker as to the philosophic significance of general relativity and whether idealism or realism prevails by virtue of it.*

A discussion of the FitzGerald contraction leads up to relativity (old style, 1905). Then the world of four dimensions (x, y, z, t) is dwelt upon, but no more than is necessary to the uninitiated. The fourth chapter discusses fields of force—the equivalence postulate. Then Kinds of Space serves as the title of a chapter which explains, as well as may be, the meat of non-euclidean geometry, the theory of curvature, the analysis of tensors. A hard chapter to write. (But we wonder whether it is true that Riemann, Christoffel, Ricci and Levi-Civita never dreamt of a physical application of their analysis (page 89). We seem to recall that Ricci, rather early, applied his absolute calculus to problems in elasticity, and Levi-Civita to problems in potential theory. Christoffel is les s known to us, but Riemann we always regarded as physicist quite as much as mathematician, and as a dreamer who dreamt many a relationship between the two.)

It may be well to ponder upon the statement (page 92): "I prefer to think of matter and energy, not as agents causing the degrees of curvature of the world, but as parts of our perceptions of the existence of the curvature." Relative to the older physics this is a turn-about quite Copernican. We have been in the habit of separating our difficulties—space, time, matter, energy, electricity, etc. The new style is to lump them all together and blame it on the way the world (four-dimensional) is made or on the way we make our measurements. It may be the better way. Only the future can tell. Certainly it does not seem the simplification that the Copernican theory represented as against the Ptolemaic. The great advances in science are usually ear-marked by simplicity,—the sort of thing the man in the street can understand if attentive. Einstein is reported to have alleged that not over a dozen in the world could read his book. That is not the way advances in science come,—but this may be

* See "Space, Time and Gravitation," by E. B. Wilson, *The Scientific Monthly*, March, 1920, especially p. 233.

the exception that proves the rule, or Einstein may merely be truthful and we may be fooling ourselves in thinking that the way of scientific advance is natural and easy, once it is pointed out by the pioneer.

Chapter VI is on the New Law of Gravitation and the Old Law. The defects of Newton's law are pointed out and the curvature of the space-time world in the neighborhood of matter is explained in detail with the aid of the differential

$$ds^2 = -dr^2/j - r^2d\theta^2 + jdt^2, \qquad (j = 1 - 2m/r),$$

of the manifold. The diagrams and analogies are suggestive and effective. Next comes a chapter on weighing light and among some excellent statements we find the puzzling one that: Possibly the tails of comets are a witness to the power of the momentum of sunlight which drives outwards the smaller or the more absorptive particles—puzzling until we surmise that probably reflection is not of much importance on account of low albedo. It is in this chapter that an account is given of the results of the eclipse observations at Sobral and Principe. Other tests of the theory form the subject of Chapter VIII,—perihelion of Mercury, minor corrections in the elements of other planetary orbits, shift of the spectral lines, and the philosophical test of the Principle of Equivalence as applied to the clock (page 131) Th s last argument is particularly interesting if somewhat dangerous, as the author seems to realize.

The chapter on momentum and energy brings in the ordinary relativist law for the change of mass with velocity But the chief interest lies in a quasi-philosophic, speculative attitude wherein gravitation and inertia are identified. If this has appeared in the writings of others, it has at least escaped our attention. There is (page 141) a suggestion that corresponding to any *absolute* property of a volume of a world of four dimensions (the property chosen by Einstein being curvature) there must be four *relative* properties which are conserved (in the Einstein theory, the conservation of energy and of momentum); and that this might be made the starting-point of a general inquiry into the necessary qualities of a permanent perceptional world. The inquiry should be made. Indeed, the statement would seem to place the Einstein theory of gravitation in the class of an astute guess for which such corroboration as the bending of light by definite quantitative amount and the correct figure for the advance of the perihelion of Mercury could not be expected. It must be possible to modify the curvature by introducing other absolute properties in conjunction with it so as to leave the major fact of (Newtonian) gravitation unaltered but seriously to alter the minor corrections to it. Action, i.e., mass or energy multiplied by time, is given a fundamental position as density multiplied by a four dimensional volume, action as the curvature of the world, and reference is (later) made to the fundamental h of Planck's quantum theory. The curvature of the world in water (density 1 gm./cm.^{3}) is the same as that of space in the form of a sphere of 570,000,000 km. radius or, in time units, of radius about 1/2 hour. A homogeneous sphere of water of this radius would exhaust all space-time. This mass is small compared with many estimates of that of the sidereal universe. Apparently, then, that mass could never get together except in a very much rarer condition than water.

"Towards Infinity" is the title of Chapter X. Here is considered the contrast between translation and rotation. Can an observer accept as a possibility a geocentric system with non-rotating earth? He would have to have something to take care of the centrifugal force $\omega^2 r$. Would a distribution of gravitating masses accomplish this? The author appears to give it up, at least as a problem in physics and to believe there is a real distinction physically if not metaphysically between rotation and translation with respect to absoluteness. (Of course on the Newtonian theory a uniform distribution of negatively gravitating matter about any center will produce a repulsive force proportional to the distance.) A few words are given to acceleration. The curved spaces of de Sitter (spherical) and Einstein (cylindrical) are discussed. The author admits that things are getting speculative and less definite than before and that he is becoming bewildered. Almost all students of generalized relativity must at times share his sensations. One phrase we quote (page 165): The reader will see how our search for an absolute world has been guided by a recognition of the relativity of measurements of physics. This seems sound.

The penultimate chapter on electricity and gravitation gives a sketch of Weyl's theory, a new geometry in which a rod taken around a circuit in space and time changes length, thereby becoming responsible for the sensations of electric shock. The theory is perplexing but it certainly has some points of philosophic advantage over Einstein's theory, besides leading to the electromagnetic equations. In it *action* is a pure number; there is so and so much *action* in a given region of space-time independent of the coordinates used or the unit of measures. The author thinks that in some way this pure number must be connected with *probability* so that the principle of least action becomes the principle of greatest probability. This is very attractive and, in the present state of our theory, very suggestive. There is a final chapter on the Nature of Things and an Appendix containing mathematical notes.

We agree with Eddington that H. Weyl's Raum, Zeit, Materie (recently appearing in a third edition) is the best treatise on the new relativity; his own is undoubtedly the best general presentation.

EDWIN BIDWELL WILSON.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY,
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Researches in Physical Optics. Part II: Resonance Radiation and Resonance Spectra. By R. W. WOOD. New York, Columbia University Press, 1919. viii + 184 pp. + X pl.

THERE is no doubt that the theory of spectra of all sorts, including resonance spectra, offers opportunity for mathematical work, both on the side of the dynamics or kinematics of hypothetical atoms or molecules and on the side of empirical nomographic or curve-fitting studies of spectral series. Considerable has been accomplished in both directions but further studies will be necessary before anything approaching satisfaction relative to our knowledge of the intimate parts of optics and of the constitution of matter is reached. Moreover, unless some extraordinary genius like Willard Gibbs appears, to do for this field what he did for physical chemistry, much additional experimental knowledge must be acquired and digested. It is this experimental foundation, with respect to resonance spectra, which Professor Wood is developing and expounding in the researches recently appearing from the Columbia University Press. The book, as written, interests the experimental and descriptive physicist rather than the mathematician, even though he be a mathematical physicist.

E. B. WILSON.