type of organization in which a steady state has apparently
been reached.   In such an aggregation, the function C must be
very nearly constant over extraordinarily long intervals of
time.   If our galaxy is such a star cluster, it is oblate rather
than globular.   Assuming that it is composed of 1.5×10^9
stars such as our sun with an equatorial radius of 2000 parsecs
(6,600 light years) and a polar radius of 600 parsecs, and that
the mean velocity of the stars is 25 kilometers per second,
the period of oscillation (if any exists) is 25,600,000 years
and the period of a circular orbit about its equator is 74,300,000
years.   If we assume a stellar density only one-fifth as great
and a stellar velocity of 40 kilometers per second instead of
25 kilometers per second, the period of oscillation is 97,000,000
years, and the period of the circular orbit about the equator is
166,000,000 years.   It should be added, however, that even
though the function C is a constant (or nearly constant) for
the galaxy, it is not necessary that there should be any oscil-
lation.   The moment of inertia may be a constant.

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SUBSTITUTIONS COMMUTATIVE WITH EVERY SUBSTITUTION OF AN INTRANSITIVE GROUP*

BY G. A. MILLER

It is well known how to find all the substitutions which are
separately commutative with every substitution of a transitive
group G of degree n and which involve no letter except such
as are found in G.   If G_1 is composed of all the substitutions
of G which omit a given letter of G, and if the degree of G_1
is $n - \alpha$, then all the substitutions on the letters of G which
are separately commutative with every substitution of G
constitute a subgroup K of order α, and all the substitutions
of K besides the identity are regular and of degree n.

When G is intransitive, the results are not quite so elegant,
but as they are often useful it seems desirable to state them

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explicitly.   It is evident that the direct product of the groups composed separately of the substitutions on the letters of the various transitive constituents of G which are commutative with all the substitutions of these constituents is a group K_0 composed of substitutions which are separately commutative with every substitution of G.   A necessary and sufficient condition that K_0 include all the substitutions on the letters of G which are commutative with every substitution of G, is that G have no two transitive constituents which are simply isomorphic with each other and in which the subgroups composed of all the substitutions which omit a letter correspond to such sub-groups.   This results directly from the following theorem:

THEOREM.   *A necessary and sufficient condition that there exist a substitution which interchanges h systems of intransitivity and is commutative with every substitution of the intransitive group involving these systems, is that there be a simple isomorphism between the h transitive groups which constitute these systems, such that a subgroup composed of all the substitutions which omit a letter of one of these h transitive groups corresponds to such a subgroup of the other.*

This theorem results almost directly from the fact that every automorphism of a transitive group in which a subgroup composed of all the substitutions which omit a given letter corresponds to such a subgroup, can be obtained by transforming the transitive group by substitutions involving only the letters of this group.   In fact, it is evident that we can establish an automorphism of a transitive group in such a way that a substitution t on the letters of the two transitive constituents thus obtained interchanges these constituents and is commutative with every substitution of the intransitive group formed by them.   By transforming one of these constituents by a substitution on its own letters which transforms it into itself the isomorphism mentioned in the theorem is obtained, and t is transformed thereby into a substitution which is commutative with every substitution of the intransitive group obtained in this way.

If G involves h transitive constituents which can be trans-

formed transitively by means of substitutions which are com-
mutative with every substitution of the group formed by
these constituents, then these constituents can be trans-
formed into each other according to the symmetric group of
degree h. The total number of such substitutions on the
letters of the h constituents can therefore be obtained by
extending the part of the direct product K_0 which relates to
these constituents by means of the substitutions of this sym-
metric group of degree h. As a very special case of this
theorem we have that all the substitutions which are com-
mutative with every substitution of the intransitive group
obtained by establishing a simple isomorphism between n
symmetric groups of degree $n > 2$ and $n \neq 6$ constitute a
group which is a conjugate of this intransitive group.*

From the preceding developments it results that all the
substitutions which are commutative with every substitution
of an intransitive group can be found by first constructing K_0
and then extending this K_0 successively by means of the
symmetric groups on h transitive constituents if any exist,
where h is always taken as large as possible. If one such h
is not equal to the number of transitive constituents involved
in G, then the group composed of all the substitutions which
are commutative with every substitution of G is a direct
product of the groups relating to the constituents which are
transitively connected. While the order of each of the con-
stituents of G which can be transitively connected must be
the same, those which can not be thus connected may evi-
dently have different orders.

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* Miller, Blichfeldt, Dickson, *Finite Groups*, 1916, p. 41. It may be
noted that the condition $n \neq 6$ should have been added there since the
symmetric group of degree 6 admits outer isomorphisms.