TWO BOOKS ON ANALYSIS


Permitting oneself a variation on a well known theme, one could perhaps say to a nation: "Show me your fundamental course in analysis and I will tell you who you are mathematically." For doubtless, a valuable estimate of a nation's state of development in mathematics can be obtained by considering the form in which the fundamentals of the infinitesimal calculus are presented to the students of that nation.

If the two works under review may be taken as representative Italian courses, one must form a high estimate of the esteem in which mathematics is held in that country. In scope, point of view and method of approach, they are broad and scholarly. They cover practically the same ground except for Pascal’s third volume which is devoted entirely to the calculus of variations and to the calculus of finite differences. They are not encyclopedic like the French cours d’analyse; neither are they written on the superficial plan of so many of our American college texts. From them a student can learn enough to be well prepared for special study in analysis, as well as for work in the applied sciences—he will at least have acquired that which he is to apply. Both intended for technical students, the authors do not hesitate to include the elements of the theory of point sets and other topics usually regarded in our classes as material unfit for the training of "practical men." In his preface, written May 1917, Pascal says “It is certain that through the profound changes which the critical spirit has made in the foundations of the calculus, even a course intended for those for whom mathematics is a means rather than an aim, cannot but use the new results which have been reached ... it would therefore exhibit a shortsighted view and little esteem for the ability of the future engineer, to believe that it would be sufficient for them, at least if they can, to learn to operate the calculus in about the way in which a workman knows how to operate a machine made by others, and of which he does not know the inner connections.”

Is not this a point of view worth the consideration of our teachers of engineering students? Not merely for the mathematical specialist, but for the person concerned with the applied sciences, a fundamental theoretical course, not merely a working course, is requisite if these applications are to be more than mere mechanical adaptations of the thoughts of others.

The two books differ in their method of treatment, inasmuch as Vivanti is in favor of and Pascal opposed to the fusion of differential and integral calculus. The _Lezioni_ of Vivanti consists of the following six parts: I. Analytical introduction (90 pp.), II. Derivatives and integrals of functions of one variable (150 pp.), III. Derivatives and integrals of func-
tions of several variables (84 pp.), IV. Geometrical applications (183 pp.),
V. Differential equations (138 pp.), VI. Calculus of variations (20 pp.).

In the analytical introduction are assembled those notions from the
theory of functions of a real variable which form the irreducible minimum
for a sound development of the calculus: real numbers, sequences and
series, greatest lower and least upper bounds, continuity, uniform con­
vergence, etc. This part reveals what an excellent expositor the author is.
The reviewer noticed with pleasure several neat proofs, such as the one
on page 39, in which it is shown that \( \lim_{x \to c} f(x) = a \) implies that \( \lim_{x \to c} \frac{f(x)}{a} = \frac{b}{a} \).

In the arrangement of the material, in the working out of details, in the
style and the form, a masterly hand shows itself. This section provides
for the reader an enjoyment which I would only spoil by dwelling on the
details.

With such preparation a thorough treatment of the calculus becomes
possible. A curious slip occurs on page 140, where from the uniform con­
vergence of a series of functions \( f_i(x) \) on an interval \( (ab) \), the conclusion
appears to be drawn that a series of functional values \( f_i(x_i) \) must converge
for arbitrary choice of the \( x_i \). It is surprising to find in this book the
notation \( \frac{df(\alpha, \beta)}{d\alpha} \) where \( \frac{df(x, y)}{dx} \), \( a \) is meant, a notation no longer
used in our better texts and, in my judgment, very undesirable. Space is
lacking for detailed mention of the many instances of elegance in treatment
found throughout these parts of the book.

It seems curious that with such free fusion of the two divisions of the
calculus, there should be such a sharp separation between the calculus
and its geometric applications, to which the largest single part of the book
is devoted. In the first 20 pages of this section we find an exposition of the
elements of vector analysis which follows the methods of Burali-Forti and
Marcolongo. Throughout the rest of the section these methods, as well
as the ordinary methods, are used in the applications to differential geom­
etry, which in many instances are improved and simplified in this way.
The incorporation of the vector treatments, assembled in an appendix in
the first edition of the Lexioni, as an organic part of the book in this new
edition, bears witness to the vogue which the work of Burali-Forti and
Marcolongo has given these methods.

In Part V, particular mention should be made of the simple and straight­
forward discussion of the singular integral and its geometrical interpreta­
tion, as well as of the treatment of the linear homogeneous equation with
constant coefficients by the use of simple relations holding between the
properties of the linear differential operator and the associated algebraic
functions. A brief section devoted to the calculus of variations treats
a few of the classical problems and merely mentions some of the others.
Numerous examples are worked out in the text. For further exercises
the reader should consult the companion volume, the author's Esercizi.*
The misprints are very few and do not seriously mar the typographical
excellence of this book.

* See the review of the book by R. C. Archibald in this BULLETIN,
vol. 20 (1914), p. 482.
In the three volumes of Pascal's *Lezioni* on the other hand, misprints are quite numerous, particularly in volume 3, where some fifty were noticed on slightly over three hundred duodecimo pages. Enough has been said above in comparing the two works to make further discussion of the first two volumes of Pascal's *Lezioni* superfluous. The third volume is little different from the first edition of 1897 of which the first part appeared in a German translation by Schepp in 1899.* It appears to be a reprint of this earlier volume rather than a new edition. It is this which accounts for the apparent disregard of the important developments in the calculus of variations during the last twenty years. It still frankly represents the old school, so that a detailed criticism from the modern point of view, for which the book furnishes ample opportunity, would clearly be out of place. The extensive bibliographies inserted at various points form a valuable feature. It is surprising, however, that in the list of treatises on page 19, apparently revised since the earlier edition, Hadamard's *Leçons* is not mentioned, while there is mention made of a *Lehrbuch der Variationsrechnung* by Carathéodory and Zermelo, which, although announced repeatedly, has not yet appeared and does not occur on Teubner's later lists of future publications.

ARNO LD DRESDEN

**SHORTER NOTICES**


The third edition of these two volumes is so nearly a reprint of the second edition, which has already been reviewed in this *Bulletin,*† that an account of its contents is quite unnecessary. The arrangement of material is precisely the same in the two editions, but many of the discussions have been slightly amplified in the later one and the few errors in printing have been corrected. The only new material appears in the derivation of curves from given properties and in the definition of the Christoffel symbols.

These two volumes should be intelligible to the student who has little training beyond the calculus, yet they present an excellent treatment of the essentials of differential geometry. The student who has read them should have no difficulty with the more extensive treatments of Bianchi, Eisenhart, Forsyth and others. Such a presentation of differential geometry as this by Kommerell and Kommerell, if available in English, would increase the teaching of that important subject to the advanced students in our American universities.

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* See the review by E. R. Hedrick in this *Bulletin*, vol. 12 (1906), p. 172.