

in instances where the usage may be regarded as classical. One is tempted to inquire, however, how these classical usages could have originated if some one had not initially honored an author by naming one of his discoveries after him? It would seem that the whole question reduces to a matter of taste, as to whether one prefers the personal or the impersonal point of view in the study of mathematical science.

C. N. MOORE

Theorie des Potentials und der Kugelfunktionen. By A. Wangerin. Band II, Sammlung Schubert LIX. Berlin and Leipzig, Vereinigung Wissenschaftlicher Verleger, 1921. viii + 286 pp.

The first volume of the above work, which deals with the fundamental portions of the potential theory, was reviewed in a previous number of the *BULLETIN* (vol. 16 (1910), p. 492). The present volume is devoted to a study of the properties of spherical harmonics and their applications to various problems of potential theory.

The book is divided into four sections. The first section deals with such properties of spherical harmonics as are essential for later developments; the second deals with potential problems for the sphere; the third with potential problems for the ellipsoid of revolution, for two spheres, and for a few other special cases; the fourth with potential problems for arbitrary closed surfaces.

The book as a whole contains more material than one would expect to find in a work of its size, and the presentation is in general clear and sufficiently refined for the purpose in hand. There are, however, certain errors and inconsistencies in the statements and demonstrations of some of the fundamental theorems that should be eliminated. For example, any one familiar with the literature on spherical harmonics will be surprised to find on page 93 the claim that the author has established the convergence of the development in Laplace's functions of any function $f(\theta, \varphi)$ that is finite, single-valued, and continuous on the whole sphere. Since examples have been constructed by Haar and Gronwall of continuous functions whose development in spherical harmonics is divergent, one naturally examines the "proof" of the so-called theorem with some curiosity. On page 89 he finds the assumption that a sum of p terms, each one of which approaches zero, will also approach zero, even though p becomes infinite at the same time that the individual terms approach zero. On page 93 he finds a statement that is equivalent to the assumption that a function continuous throughout a certain interval does not have an infinite number of maxima and minima in that interval.

It would surely be better to omit entirely any proof of the development theorem, important as it is for the subsequent theory, than to introduce such obvious errors with regard to the fundamentals of analysis in the course of the demonstration. A less radical remedy is available, however, for by adding a further restrictive condition to the statement of the theorem and a relatively small amount of material to the proof, the whole discussion could be put on an entirely rigorous basis. It is to be hoped that these changes will be made, if a subsequent edition appears.

C. N. MOORE