HEATH ON GREEK MATHEMATICS

A History of Greek Mathematics. By Sir Thomas Heath. Volume I, *From Thales to Euclid;* volume II, *From Aristarchus to Diophantus*. Oxford, Clarendon Press, 1921. xvi + 446 + xii + 586 pp.

It is doubtful if anyone could assume the privilege of writing a review (at least an American review) of this monumental treatise without a feeling of helplessness. An American review must, by our present standard of taste in matters literary and critical, be brief, be popular, and (according to certain other standards) be both flippant and faultfinding; but no brief review can do this work justice nor can a sufficiently extended review be popular, while either flippancy or faultfinding would be merely a display of poor taste, like loud boasting or any other of the various species of vulgarity.

With such thoughts, a reviewer may properly ask himself what, precisely, is his mission. Is it to seek out points of doubt in the narrative and magnify them into blunders, or to search out minor inaccuracies (for such can always be found in any book) and display them as typical of the work under inspection? Should we take seriously Montaigne's *mot*, "Since we cannot attain to greatness, let us have our revenge by railing at it"? Or has the critic a more wholesome duty to his readers—that of stating the salient features of an author's work and of discovering whether he has seen things as they really are, has recorded them with becoming felicity of style, and has searched out such of the causes for world progress as can be shown to exist?

In considering the work under review, therefore, it is proposed to ask whether the author has seen Greek mathematics as it was, and whether he has expressed the results of his studies in a style that will command the willing attention of those whose tastes will lead them to read a work of this nature. In the seeing of Greek mathematics as it was, there is involved the question of causes, of the general setting of mathematics in the philosophical schools of that remarkable race, and of the balancing of the merits of different investigators and expositors. In the matter of style, a subject admitting of only slight mention, there may well be raised the question of the influence upon the author of the Greek language in which the science was expressed—a subject not without value in these days in which Greek is reported to have become obsolete as a subject of study in the schools of our country.

First, then, has the author seen the Greek mathematicians as it were face to face, intellect to intellect, as one master of the subject to another? —or has he seen them as in a glass, darkly, trusting to secondary sources and to unexplored tradition? The answer is a simple one—that no man now living is more capable than he of interpreting the Greek mathematical mind to the scholar of today; indeed, there is no one who ranks even in the same class with Sir Thomas Heath in this particular. Paul Tannery might have done so in his lifetime, but in knowing the mathematics of Greece, in knowing the Greek of mathematics, and in knowing the causes which led to the great development of the science in Miletus, in Crotona, in Athens, in Alexandria, and in the islands of the Ægean Sea, the author of this treatise stands without a rival, and these causes he has set forth for us in his usually lucid style.

This is a strong assertion, it sounds exaggerated, and it has the odor of that type of flattery from which Byron and many another of lesser fame has begged to be delivered—and yet the assertion is perfectly true. Sir Thomas Heath ranked in his university as a leader in mathematics and as a leader in Greek, and thus he was equipped as few others have been to see Greek mathematics with an intimacy that most men have been unable to enjoy, and he has set forth the results of his intimate knowledge with a felicity of expression that could come only from familiarity with the language of the men whose works he has described.

The first difficulty that the historian meets in making known the results of his studies is analogous to the laying out of the ground plan of a structure; it requires him to consider which of several designs he will take and what shall be the order of rooms through which the visitors to the edifice shall be conducted. Moreover, in the case of the history of any science a writer is confronted by one particularly serious difficulty, namely, that of the sequence of chapters. A reader may properly demand that the treatise present the growth of the subject in chronological order, and that it shall also present a subject like analytic geometry as a unit. These two demands are, however, mutually antagonistic, since the chronological order of a history of mathematics would scatter the evidences of the development of any of the leading topics along a period of two or three thousand years, while any mere topical treatment would require a wearisome repetition of chronological and biographical material with every topic considered. Historians are continually trying to harmonize these methods, even as astronomers have tried from time immemorial to harmonize the lunar and solar calendars, and the result in each case is necessarily a compromise. The author has himself called attention to the difficulty, characterizing Professor Loria's *Le Scienze esatte nell' antica Grecia* as "the best history of Greek mathematics which exists at present," and showing that the distinguished Italian historian had taken, as he frankly states, "a compromise between arrangement according to subjects and a strict adherence to chronological order, each of which plans has advantages and disadvantages of its own."

In the work under review the author has made the attempt to solve the problem by arranging his chapters as follows:

I. *Introductory*, the purpose being to give the reader a general view, as from an airplane, of the terrain through which he is to be led—a desirable preliminary treatment in any historical treatise.

II. *Greek numerical notation and arithmetical operations*, setting forth what the reader should know of that part of the numerical art which the Greeks called "logistiké."

III. *Pythagorean arithmetic*, in which is given in considerable detail a survey of that which was known as "arithmetikê"—the theory of numbers.

IV. *The earliest Greek geometry*, in which are set forth the life and labors of Thales. It will be noticed that in Chapters II–IV the author is forced to abandon a strictly chronological sequence, Chapter II considerably overlapping Chapter III, and each overlapping Chapter IV. This is a necessity for a writer who attempts a topical arrangement, and it results in this case in the omission from Chapter III of the contributions of Euclid and Diophantus, for example, these being considered later in chapters devoted to the men themselves.

V. *Pythagorean geometry*, in which the story of geometry is continued, but in which the difficulty is necessarily encountered of distributing the biography of Pythagoras between Chapters III and V. It is interesting to observe that the author has less to say of the life and times of this great philosopher than of any other leading Greek mathematician, probably because less authentic material is available for a biographical sketch. Concerning the much mooted question as to the source of the first demonstration of the theorem which bears the name of Pythagoras, the author gives a judicial summary of the evidence and concludes with the statement: "I would not go so far as to deny to Pythagoras the credit of the discovery of our proposition; nay, I like to believe that tradition is right, and that it was really his"—a decision that will meet with the approval and command the respect of the great majority of students of history.

VI. *Progress in the Elements down to Plato's time*, in which a study is made of one of the most interesting periods in the development of Greek geometry—the formative stage in which proofs were discovered and the logical bases of the science were beginning to be sought. It is now possible for the author to give to the treatise an arrangement that is more nearly biographical, and to set forth the biographies in chronological sequence.

VII. *Special problems*, in which the "three famous problems" of antiquity are considered. Here, as in Chapter VI, the nature of the subject permits of the biographical and chronological treatment. Among other details, Bryson's contributions to the study of the method of exhaustion are recognized more favorably than has of late been the case.

VIII. *Zeno of Elea.* It is probably quite justifiable to give Zeno a chapter by himself, since it would be difficult to place him with anyone else. While the author has not carried his study of the history of the philosophic interpretation of Zeno's problems as far as our Professor Cajori (to whose contribution he pays just tribute), he condenses in a few pages the best of the Greek interpretations of his paradoxes.

IX. *Plato*, in which chapter there is given a succinct statement of the influence of this great philosopher with respect to the foundations upon which a work like Euclid's should rest, this statement being fortified by extracts from Plato's works as well as from those of subsequent writers who were conversant with his doctrine.

X. *From Plato to Euclid*, the period in which the post-Pythagorean accumulation of propositions and the influence of Plato with respect to foundation principles were working towards the making of a treatise which should set forth the Greek geometry in all its excellence. In this period falls the work of Aristotle, not generally enough appreciated in its influence upon geometry but here given just recognition.

XI. *Euclid*, whom no one in our time has recognized so worthily as Sir Thomas Heath, and to whom has here been given about one fourth of Volume I. Naturally the author has done little more than condense into about a hundred pages his own monumental treatise upon the *Elements*, using much of the language there employed, and he could not have done better than follow this plan. Since the belief is not uncommon that Euclid was merely a textbook writer, devoid of mathematical genius, the tribute here paid, showing his genuine powers as a geometer, is welcome. It has been said of Shakespeare that he "took the stillborn children of lesser men's brains and breathed on them the breath of life," and Euclid may have done the same, but it takes a genius, perhaps a divine genius, to perform this miracle.

XII. *Aristarchus of Samos*, more of an astronomer than a geometer, but nevertheless one of the first great geometricians in the astronomical field. It is characteristic of the author that he makes no mention of his own treatise on Aristarchus, indeed, that he hardly refers to any of his other works. This is an illustration of British modesty, to the lack of what they call "side," which we in America (probably unfortunately) fail to understand; for not infrequently the reader might be assisted by more frequent reference to such standard works as those which the author has contributed to the study of Greek mathematics.

XIII. *Archimedes*. It is a tribute to five of the greatest names in the field of ancient mathematical research and exposition that the author has given to each approximately a hundred pages, the total amounting to about half of the entire treatise. These men are Euclid, Archimedes, Apollonius, Pappus, and Diophantus. Of these it is an easy matter to pick out the least, but it is difficult to select the greatest. Probably, if a ballot were taken among those who have the intellectual right to vote, the choice would fall upon Archimedes, and the treatment which the author has accorded him is in harmony with this judgment.

XIV. *Conic Sections: Apollonius of Perga*. As in the cases of Euclid, Aristarchus, Archimedes, and Diophantus, the intellectual world is already familiar with the author's treatises upon this great expositor (and doubtless largely the creator) of the ancient theory of conics. There is no better way of securing an insight into the essential difference between the mathematics of the Greeks and that of the present day than by comparing the treatment, say of the ellipse, as given by Apollonius, with that given in our own modern textbooks, and perhaps there is nothing that gives a student a higher appreciation of the Greek mind. For one who wishes this opportunity but who has but little time for the comparison, this chapter will prove especially helpful.

XV. *The successors of the great geometers*, in which chapter is given a brief statement of the work of those who began, in any large way, the theory of higher plane curves—a theory which Greece had no longer the intellectual strength to complete. The names considered include those of Nicomedes, Diocles, and Perseus—each of whom is known chiefly for a single important contribution to geometry.

XVI. *Some handbooks*, under which properly depreciative title there lie the works of Cleomedes, Nicomachus, and Theon of Smyrna. If ever a man was pushed into fame in the field of mathematics, by little more than chance, that man was Nicomachus.

XVII. *Trigonometry: Hipparchus, Menelaus, Ptolemy*, in this order, for "the first person to make systematic use of trigonometry is, so far as we know, Hipparchus." It was with him that the long union of trigonometry and astronomy began, a union particularly noticeable in the Arab schools, and only broken when each science, in the fifteenth century, had so developed that it was able to stand alone and to set up an establishment for itself.

XVIII. *Mensuration: Heron of Alexandria.* Aside from setting forth a summary of Heron's work the author devotes considerable attention to the controversy which has so long been waged as to the time in which this great scholar lived. Not even Diophantus has given historians so much trouble, partly due to the fact that there were a number of Herons whose names have come down to us together with some knowledge of their achievements. Until recently the Heron of mathematical fame has been commonly placed in the first century B.C. (and sometimes earlier), but of late he has been put in the first century of our era. Heiberg felt that the third century would be a safer conjecture, and Sir Thomas Heath, after carefully weighing the evidence, thinks the same. The evidence is by no means conclusive, but it seems certain that the date was about 50 A.D. or not more than two centuries thereafter.

XIX. *Pappus of Alexandria*, the last of the prominent geometers of Greece; not a great genius but, considering the time in which he lived, a great scholar and a worthy mathematician.

XX. *Algebra: Diophantus of Alexandria*, much more of a genius than Pappus or Heron, a great mathematician in a period of general scientific decay, and the one who best deserves the title of "father of algebra."

XXI. *Commentators and Byzantines.* Under this rather interesting caption, with its hint at mutual exclusion, are considered the names and works of men like Serenus, Theon of Alexandria, Proclus, and Psellus, and of the only prominent woman mathematician of all antiquity—Hypatia. It was the period of the death of Greek science, and, as with all such periods, its chronicles are not stimulating to the mind.

The second volume has a good index and a list of Greek terms.

The limits set for reviews at the present time are such as to allow no adequate statement of the merits of this noteworthy treatise. It is only possible to add that it is destined to be the standard work upon the subject, even as the author's other works are the recognized standards in their respective domains.

The feature which, with respect to the substance of the text, distinguishes the work from any other of its kind is the large amount of source material that it contains. The author has not merely written a history; he has set forth at unusual length the evidence to support his views. Thus we have extracts generously made from such writers as Archimedes, Apollonius, Pappus, and Diophantus by which the reader is able to form a first-hand opinion of the nature and value of their several contributions and methods of attack. Such a plan operates against a literary production of uniform smoothness of expression, and it considers the needs of the scholar instead of the taste of the casual reader; but it is precisely the scholar whom the author has in mind rather than a more general intellectual public for which men like Gibbon and Guizot wrote their justly celebrated treatises. It is difficult to accomplish both purposes; Libri attempted it by giving the scientific world a text composed with true French elegance and supplementing this by source material in an appendix, but in so doing he made the necessary sacrifice of any close connection between his text and his evidence, and moreover his selection of material was rather on the basis of rarity than of importance. On the whole, indeed, no one has solved the problem more effectively than has been done in the work under review.

It should also be said that the plan of quoting so fully from the ancient writers serves to remove any doubt as to the validity of the author's thesis that "the foundations of mathematics and a great portion of its content are Greek. The Greeks laid down the first principles, invented the methods *ab initio*, and fixed the terminology. Mathematics in short is a Greek science, whatever new developments modern analysis has brought or may bring." To establish such a thesis there is demanded not merely the assertions of today; we must have the precise evidence of the past, and this is what has been placed before the reader in such abundance as to make the work a source-book as well as a historical narrative.

If it should be asserted, as the reviewer has already heard it remarked, that the arrangement of material suggests a set of essays rather than a connected historical discourse, the obvious reply is that the history of Greek mathematics is largely a record of the work of a few great geniuses; that it is not, like political history, a list of innumerable wars and changes of dynasties and of perpetual slaughter and rapine, nor even like economic or social history, and that it therefore requires precisely the treatment here accorded to the story of the leaders of scientific thought. It is idle to speculate as to whether the preliminary historical essay might have been expanded into a volume, and the biographical material have been there disposed of first, the special topics being placed in the second volume. Suffice it to say that the author felt that he could accomplish his purpose better by pursuing the route he has taken, and every writer should take the road he can travel most safely. The goal has been reached, the way has been delightful, the guide has spoken as one having authority, and the scientific world should be accordingly generous in its praise and in its appreciation and its thanks.

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