SHORTER NOTICES

Théorie Mathématique des Assurances. By P. J. Richard et E. Petit. Second edition, revised throughout and brought up to date. Paris, Librairie Octave Doin, Gaston Doin, Editeur, 1922. Volume I, 455 pp.; Volume II, 320 pp.

These two volumes appear in the series entitled ENCYCLOPÉDIE SCIENTIFIQUE–BIBLIOTHÈQUE DE MATHÉMATIQUES APPLIQUÉES. The first volume, book one, contains two chapters on the theory of probability, with development of elementary theorems, the theorem of Bernoulli, Baye's rule, and a consideration of insurance from the point of view of a game of chance.

Book two contains a chapter on life and mortality tables and gives definitions of the rate of mortality and derived functions. The effects of age, sex, occupation, climate, selection, and other causes on the rate of mortality are explained. Various methods of constructing and graduating the rate of mortality are described, including graphical, mechanical and analytic adjustment.

Several proposed laws of human mortality including the formulas of De Moivre, Gompertz and Makeham and other generalizations are given. The old and the modern French tables for annuitants and for insured are described together with methods of adjustment and graduation to Makeham's law.

Chapter II is divided into four parts. The first part relates to annuities-certain and life annuities and develops the general theory of various types of annuities and their expression in terms of commutation symbols. Part two deals with last survivor annuities and annuities to x after the death of y. The theory of single premium insurance benefits on one life and on joint lives is developed in part three and the last part deals with a variety of forms of insurance and gives methods of deriving the net single and annual premiums.

Chapter III consists of five parts devoted respectively to gross premiums and methods of loading, mathematical reserves, contracts of insurance and alterations, dividends in insurance, and concludes with a study of the mathematical insurance risk and the stability of insurance based on the theory of risk.

This volume concludes with a very good six page bibliography of life insurance literature.

The second volume is divided into two books. The first book is devoted to insurance on the individual and consists of five chapters.

The first chapter develops very fully the fundamental probabilities in the theory of insurance against disability, the construction of disability tables, of rates of mortality among the disabled, and so on. A brief account is given of published statistics and of disability tables in use. It is curious that no mention is made of Hunter's *Disability Table*, the one most used in this country.

The second chapter contains an explanation of formulas for disability premiums and reserves under various conditions.

In chapter III there is given an account of the theory and derivation of rates of sickness, of the statistics pertaining thereto, the principal sickness tables, and the methods of calculating net and gross sickness premiums and the mathematical reserves.

Chapters IV and V have to do with applications of the combination of insurance against sickness, disability and death, and also a variety of other forms of insurance relating to marriage, birth, civil responsibility, and so on.

Book II, which deals with collective insurance, gives the technical basis of social insurance and special treatment of insurance against travel accident. An appendix contains an account of the operation of the Caisse Nationale for insurance against death, accident, and to provide for old age. It contains also a brief history and treatment of tontines and societies operating on a tontine basis, and an account with formulas of certain financial operations on a collective basis.

The second volume closes with seven pages of bibliographic references of much value to students of the subjects mentioned therein.

In the opinion of the reviewer these two books cover an unusual variety of insurances, especially the second volume, and the clear and complete development of theory must meet with general approval among mathematicians interested in these subjects. The practical applications are taken chiefly from foreign sources and have little bearing on the methods used in this country.

J. W. GLOVER

Leçons sur les Invariants Intégraux. By E. Cartan. Paris, Hermann, 1922. x + 210 pp.

The theory of integral invariants was first set forth by H. Poincaré in the third volume of *Les Méthodes Nouvelles de la Mécanique Céleste.* Cartan develops this theory systematically in his Leçons, using a different point of view. Instead of connecting an integral invariant with a system of differential equations as did Poincaré, Cartan considers the integrand of this integral as a differential form and studies its properties of invariance under a group of transformations.

The mathematician will be especially interested to read this book because of the elegant treatment of the subject. Enough of the classical theories of differential forms and continuous groups, for instance, is given to make the book readable. Considerable space is devoted to the development of methods of deriving integral invariants, such as Jacobi's last multiplier and infinitesimal transformations.

These Leçons are to be recommended to students of applied mathematics because of the admirable treatment of physical problems from the modern point of view. The theory of turbulence, the n-body problem, and certain problems in optics are considered from the point of view of tensors and their relation to the integral invariant theory.

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