THE APRIL MEETING OF THE SAN FRANCISCO SECTION

The forty-first regular meeting of the San Francisco Section of the American Mathematical Society was held at Stanford University on April 7. Professor Cajori presided. The total attendance was twenty-three, including the following seventeen members of the Society:

Alderton, Allardice, Bernstein, Blichfeldt, Buck, Cajori, A. F. Carpenter, Growe, Hoskins, Lehmer, Levy, Libby, McEwen, F. R. Morris, Smail, Pauline Sperry, A. R. Williams.

The Secretary read a communication from Secretary Richardson asking the members of the Society to cooperate with the Endowment Committee, headed by Professor Coolidge, in the forthcoming campaign for funds for the Society.

In response to a request by a committee from the University of Washington for more active participation in the work of the Section by members of the Society living in and about Seattle, it was decided to hold a special meeting of the Section at Seattle in December, 1923, with Professor Carpenter acting as Secretary.

It was decided to hold the next regular meeting of the Section at Los Angeles, September 17–20, in connection with the meeting of the Pacific Division of the American Association for the Advancement of Science.

Titles and abstracts of papers read at this meeting follow. The papers of Professors Bell and Neikirk were read by title.

1. Professor E. T. Bell: *Euler algebra.*

This name is given to the algebra of certain infinite processes fundamental in the study of the multiplicative properties of numbers. Two other algebras, called here the Cauchy and Dirichlet varieties, are also investigated. The first is that of power series, the second that of Dirichlet series. Euler algebra is a species of resultant between these two. The paper is to appear in the TRANSACTIONS OF THIS SOCIETY.

2. Professor E. T. Bell: *A class of numbers connected with partitions.*

The theory of partitions and that of the representation of an integer as a sum of squares originating in the elliptic modular functions must be closely interconnected. The connections depend upon eight new systems of integers. These integers are functions of two parameters. For values $\not> 6$ of the parameters the numbers are connected with the class numbers of binary quadratic forms of a negative determinant. The paper will be published in the AMERICAN JOURNAL.

3. Professor E. T. Bell: *Square-partition congruences.*

This paper will appear in full in an early issue of this BULLETIN.

4. Professor E. T. Bell: *Theta functions and arithmetic.*

Any relation whatever between theta functions of $p > 1$ arguments implies and is implied by a corresponding relation between the solutions of a set of $\frac{1}{2}p(p+1)$ indeterminate equations of the following sort: each equation in the set is restricted only in that its coefficients must be (arbitrary) integers; the equations are not necessarily homogeneous; the degrees of the several equations in the set may be any $\frac{1}{2}p(p+1)$ positive integers, and likewise for the numbers of indeterminates. The excepted case $p = 1$ is treated in a forthcoming paper in the TRANSACTIONS OF THIS SOCIETY.

5. Professor E. T. Bell: *Note on total representations as sums of squares.*

If $O(r)$, $E(r)$ denote respectively the total numbers of representations of r as the sum of an odd, an even number of squares with roots different from zero, $O(2n+1) > E(2n+1)$, $E(2n) > O(2n)$ for all positive integers n.

6. Professor B. A. Bernstein: *Complete set of representations of two-element algebras.*

The author gives simple arithmetic representations of all two-element abstract algebras, with some applications to fundamental problems connected with postulate-sets.

7. Professor H. F. Blichfeldt: *Note on quadratic forms.* Preliminary report.

Given a positive quadratic form f of determinant D in n variables. Integers, not all zero, may be substituted for the variables in this form such that the numerical value of f is $\leqq \gamma_n \mid D \mid^{1/n}$, where γ_n is a function of n only. For $n < 6$, the lowest permissible values of γ_n have been found; γ_6 was proved $< \sqrt{3}$ by Korkine and Zolatareff in MATHEMATISCHE ANNALEN, vol. 6 (1873), p. 378. The present author proves that

$$\gamma_6 < \{8\sqrt[4]{72}\}^{1/6} = 1.690\cdots.$$

8. Professor Florian Cajori: *Varieties of minus signs.*

It is shown that besides the sign $-$ there was introduced, in 1525, $\div$ as a minus sign. This maintained its place in a few mathematical books until the latter part of the nineteenth century. Other minus signs found in books are $\dot{\div}$, $--$, $---$, $\sim$.

9. Professor Florian Cajori: *The evolution of the modern exponential notation.*

The advance toward our modern exponential notation made by the forerunners of Descartes, namely by Chuquet, Bombelli, Digges, Cataldi, Stevin, Romanus, Girard, Bürgi, Reymer, Kepler, Schoner, Hérigone and Hume, is traced with fullness of detail; also the spread of the cartesian symbols and of Newton's extension of it.

10. Professor A. F. Carpenter: *Flecnodal properties of a ruled surface.*

This paper has to do with certain quadrics connected with the flecnode and complex curves of a ruled surface, together with four related space cubics. The theorems deduced involve projective properties of these curves and surfaces, the method of attack being that of the projective differential geometry. The paper will appear in the TÔHOKU JOURNAL.

11. Professor D. N. Lehmer: *On congruences connected with magic squares with odd number of cells.*

The familiar *uniform step* method of constructing odd magic squares is based on certain interesting congruences which seem not to have been studied. One can tell by means of these congruences whether a square constructed with a given *step* and a given *breakstep* will be magic, and whether it also will be *diabolic* and *symmetric.*

12. Professor L. I. Neikirk: *Some non-associative linear algebras.*

The units of these algebras are transformations in a Galois field, and are finite in number. Two kinds of third and higher powers of units may exist, according to the way the factors are associated, and may be designated as right and left handed powers. Division is not always possible; a unit may not have either a right hand or a left hand inverse.

13. Professor L. L. Smail: *Some theorems on uniform convergence of infinite products.*

Using theorems by Bôcher and Birkhoff on the uniform convergence of $\sum u_n$ and $\sum |u_n|$ (ANNALS OF MATHEMATICS, vol. 4, p. 159, vol. 6, p. 90), a set of theorems are developed giving relations between $\Pi(1 + u_n)$, $\Pi(1 + |u_n|)$, $\sum u_n$, and $\sum |u_n|$.

14. Professor L. L. Smail: *Note on derivatives of a vector product.*

An interesting and simple formula for the nth derivative of the cross- or vector product of two vectors is obtained.

15. Professor L. L. Smail: *Report on a synopsis of the theory of summable infinite processes.*

The author reports that he has in preparation a book on *Synopsis of the Theory of Summable Infinite Processes*, which is to cover exhaustively the field of summable series and other processes, by presenting without proof all the important results, definitions and theorems thus far obtained. This would do for the subject of summability what Dickson's *History of the Theory of Numbers* has done for that subject.

16. Dr. Pauline Sperry: *On real elements in imaginary linear spaces.*

In this paper the author shows that all real points in an imaginary linear k-space lie in a real linear $(k - 1)$-space, also that this real $(k - 1)$-space exists if and only if the imaginary k-space lies in a real $(k + 1)$-space, and obtains the theorems corresponding to these by duality. A number of theorems follow concerning necessary and sufficient conditions for the determination of real spaces by sets of imaginary elements.

B. A. BERNSTEIN,
Secretary of the Section.