*Grundzüge der Mehrdimensionalen Differentialgeometrie in direkter Darstellung.* By D. J. Struik. Berlin, Springer, 1922. 198 pages.

In addition to the research papers which are published in any subject, there comes a time when one feels the need of a book which unifies the whole. The differential geometry of hyperspace has reached this point. Practically the only books on the subject are Killing's *Nichteuklidische Raumformen* and chapters in Bianchi's *Geometria Differenziale* so that the appearance of a book devoted entirely to differential geometry of $n$-dimensions is most welcome. This subject can well be said to have started with Riemann, followed by Christoffel, Beltrami, Lipschitz, and others who devoted a great deal of time to the study of the quadratic differential forms in $n$ variables. These works were published between 1860 and 1880. In the latter eighties, Ricci and Bianchi began their work on $n$-dimensional geometry, and they and their followers have continued the work to the present time.

Ricci began his work in the absolute calculus in 1887, and he and Levi-Civita developed the subject almost to its present completeness many years ago, but it attracted but little attention. In fact it might be said that their work was practically unknown prior to 1913. Bianchi does use the covariant derivative as a notation but gives no indication of the powerful tool which Ricci made of it. However, when Einstein wrote his theory of gravitation and used the absolute calculus in its development, the subject became a live one and since that time there has been a whole army of mathematicians, scattered all over the world, working on it. The name which Ricci used was changed by Einstein to tensor calculus, and many people today are almost in total ignorance of what Ricci and Levi-Civita have done. It was, therefore, pleasant to find that Struik has dedicated his book to Ricci.

One of the outstanding features of the book is the "direct development". By introducing a sort of vector notation his work is freed from the dependence on the particular coordinate system used. The multiple algebra of the subject is developed to some extent but when one finds so many products used, it is quite a task on the memory of the reader to keep them all in mind. The notation, however, does allow one to sidestep the great mass of summation signs used by Ricci. The Clebsch-Aronhold notation is used to write forms of higher degree than the first as symbolic products of forms of the first order. Ricci's term "system" is replaced by "Affinor", and tensor is defined as a symmetric affinor. The first chapter is devoted to the multiplication of "affinors" both covariant and contravariant and mixed. In the second chapter the covariant derivative is developed and applied. The notion of geodesic parallelism introduced by Levi-Civita is made the basis of the work of this chapter. A differential is defined which bears the



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same relation to an ordinary differential that geodesic parallelism bears to ordinary parallelism and from this the operator $\nabla$ is defined similar to the usual definition. The absolute derivative is then derived as $\nabla V$ where $V$ is a vector or system of the first order. The various products of $\nabla$ with the various affinors give the formulas needed in hyperspace. Chapter three deals with the curvature properties of a curved space of $m$ dimensions contained in a curved space of $n$ dimensions which do not depend on the Riemann-Christoffel tensor. The whole development is similar to the ordinary treatment of an $m$-spread in a euclidean space of $n$ dimensions. There is a remarkable similarity in the formulas in the two cases.

The fourth chapter treats those curvature properties which depend on the Riemann-Christoffel tensor.

The text is preceded by an introduction of twelve pages which it is worth anyone's while to read. The last twenty pages are devoted to a bibliography which contains nearly four hundred titles. This gives one a good notion as to the historic development of the subject. The author has been very careful to give credit to the proper author for all ideas and formulas. The reference one meets most frequently is to Schouten, for the notation used is largely due to him.

The book is not easy reading, but as one becomes more familiar with the notation, he will find that the difficulty decreases, and he will feel amply repaid for his trouble.

C. L. E. Moore

*A First Course in Nomography.* By S. Brodetsky. London, G. Bell and Sons, Ltd., 1920. 135 pp.

Brodetsky has supplied us with the sort of book the subject has needed: a brief, readable exposition of elementary character. It will well serve two purposes. It is an excellent introduction to D'Ocagne, and it also enables its reader to gain a practical working knowledge of the nature and uses of nomograms with a minimum expenditure of time.

The author states in his preface that "it is the object of this First Course to offer a clear and elementary account of the construction and use of such (nomographic) charts" and that "it is a treatment that should be found useful by the reader who desires to become acquainted both with the theory of nomography and with its practical use."

On the whole the author has carried out his intentions fairly well, but, as a text for college students, the book is open to criticism. While a considerable amount of knowledge of algebra, trigonometry, and analytic geometry is presupposed, the author's treatment is very uneven in its demands upon the reader's knowledge and intelligence.

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