# SHORTER NOTICES

*Vorlesungen über die Theorie der Algebraischen Zahlen.* By E. Hecke. Leipzig, Akademische Verlagsgesellschaft, 1923. 264 pp.

This book had its origin in lectures delivered by the author in Basel, Göttingen, and Hamburg. As stated in the preface the aim of the book is to bring the reader to a comprehension of the questions which at present form the summit of the theory of algebraic number fields, without presupposing any knowledge of the theory of numbers. In reading the book the reviewer was particularly impressed by its richness in content. By the skillful coordination of important notions, the author gives, in the 264 pages, an elegant and comprehensive account of the modern theory of algebraic numbers.

The first chapter contains an introductory account of the theory of rational numbers. This is followed by a chapter of 28 pages on the theory of groups with special consideration of abelian groups, both finite and infinite. The purpose of this chapter is to familiarize the reader with those properties of abelian groups which may be advantageously used in the further development of the theory of rational numbers as well as the theory of algebraic numbers.

In the third chapter the group properties developed in the second chapter are used in the further study of the rational numbers. The chapter contains the theory of solutions of congruences; the theory of power residues; and in particular that of quadratic residues leading up to a statement of the law of quadratic reciprocity. The proof of this law is, however, deferred to Chapter eight, which contains the author's proof of the general law of quadratic reciprocity in a general algebraic number field.

Chapter four contains the algebraic development of the theory of algebraic numbers, and the algebraic properties of fields defined by the roots of algebraic equations.

Chapter five is the longest chapter of the book and contains a very comprehensive account of the arithmetic of an algebraic number field. The general theory of ideals is developed and theorems regarding the factorization of rational primes are given for certain special types of fields. This is followed by Minkowski's theorem on linear forms, with its application to the study of the units of a field.

A brief discussion of rings is given, and by the use of fractional ideals the author develops that part of the theory which deals with differents and discriminants of fields, and conductors of rings.

The sixth chapter is an introduction to the use of transcendental methods in the theory of algebraic number fields. It contains the development of the expression for the density of the prime ideals of a class, and its application to the problem of the determination of the number of classes. This is followed by a consideration of the zeta function and some of its applications.

Chapter seven is devoted to a more detailed consideration of quadratic fields. Norm residues are considered and the properties of Hilbert's norm residue symbol are developed. The separation of the ideals into genera is studied and the number of genera determined.

The further topics considered in this chapter are the zeta function in a quadratic field; the expression for the class number in a quadratic field (this without the use of the zeta function); the Gaussian sum; and the relations existing between ideals of a quadratic number field and binary quadratic forms.

Chapter eight contains a new proof of the most general law of quadratic reciprocity in an arbitrary algebraic number field. In this development the author makes use of his generalization of the Gaussian sum.

The chapter ends with a proof of the existence of class fields of relative degree two. This is a consequence of the general law of reciprocity.

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*Die Philosophischen Grundlagen der Wahrscheinlichkeitsrechnung.* By E. Czuber. No. XXIV of Wissenschaft und Hypothese. Leipzig and Berlin, B. G. Teubner, 1923. 8 + 343 pp.

Czuber's book on the philosophy of probability is in essence a clarification of the fundamental concepts and theorems of the theory of probability by means of analysis and criticism of attacks directed against the theory in certain recent writings. The author's patient analysis and kindly criticism center chiefly on Meinong's rather labored *Über Möglichkeit und Wahrscheinlichkeit* and Marbe's naive ideas on probability expressed in his *Die Gleichförmigkeit in der Welt.* The concept and definition of probability, the addition and multiplication theorems, the theorems of Bernonlli, of Poisson, and of Bayes, the relation of probability to our knowledge of nature, all come in for detailed discussion.

Czuber's book, in the opinion of the reviewer, is intended primarily for non-mathematicians. Students of mathematical probability will find in the author's admirable *Wahrscheinlichkeitsrechnung* (3d edition, 1914) the substance of nearly all his views on the fundamentals of probability found in the *Grundlagen.* One instance of departure from his earlier work may be worth noting here: the obtaining (in Chap. VI) of the theorem of Bayes as a simple corollary of the multiplication theorem for dependent events (compare Boole's *Laws of Thought*, p. 254, III).

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