tures of Bernoulli polynomials and gamma functions he has listed only the most important works. The bibliography is a very useful one. It is hardly to be expected that it should be complete. In fact I have found a considerable number of omissions by checking it against the partial bibliography which I have collected in an incidental way during the past fifteen years.

It is natural to expect that an exposition of a general subject should involve an important element depending on the personal interests of the author; and this is particularly true in the case of a book which is essentially the first in its field. But in the present book this element appears to me to have played too large a role in determining the distribution of emphasis and the selection of material. Much of the work in the first two hundred pages might well have been given with less fullness and the space so gained have been utilized in the presentation of some of the important matters which are omitted.

While this book will probably stand for some time as the best book in its field, and as such is therefore of great importance, it can not be regarded as having come near to being a definitive treatise on the difference calculus, even in its present state of development. Whatever one may think of the distribution of emphasis and selection of material in this volume there is still a definite need for another book with a quite different distribution and selection — one in which the personal equation of the author does not play so large a role.

R. D. Carmichael

ENRIQUES ON ALGEBRAIC GEOMETRY

Lezioni sulla Teoria Geometrica delle Equazioni e delle Funzioni Algebriche. Vol. I and vol. II. By F. Enriques. Bologna, O. Chisini, 1915, 1918.

The reviewer, far from being a specialist in algebraic geometry, commences this short review with the misgiving that in the last two years of reading "at" the work of this famous author he has set his wisdom teeth into a sticky mouthful. It may however be said at the outset that the paucity, however unfair, of references to living workers in American universities,—a general reference to Osgood's *Funktionentheorie* and particular ones to Scott (C. A. Scott) and "Angas" ("Ch. Angas Scott"(!)),—indicates that the field is one in which few of us are expert, and therefore not only that the point of view of the reviewer will be that of most of his readers, but also that the treatise itself unbars a field which some of us might well explore. The reviewer tried the experiment of lecturing from it to capable, and patient, advanced students during the past academic year.

In the first place, although much elementary material is given, the prerequisites in the way of geometry and algebra are not meagre. Since many particular theorems of projective geometry are used and since the connection between algebra and geometry is obtained mainly by means of projective coordinates, which are independent of metrical axioms, the book should be preceded by a good study of projective geometry, such as Enriques's own, to which many references are given. The reviewer found to be even better adapted for this purpose the simpler logical structure of Veblen and Young, volume I and parts of volume II, of which he gave a three months analysis in the lectures just mentioned. On this basis, for example, the theory of linear systems and abstract elements in Book I is much simplified. As to prerequisites in algebra, one may assume that for advanced students such things as linear dependence and some ideas about substitutions and groups will be familiar; but can one say the same about topics like resultants and discriminants of functions of more than one variable and the Galois theory? The reader will save time by knowing something of these in advance. Perhaps finally, for good measure, he might have at hand Miss Scott's book as an introduction to the second volume and a general indication of the sort of problem to be considered.

The general plan of the treatise, in the author's words, is to investigate all those methods and concepts suggested originally by a study of plane algebraic curves, and to apply them in such a way as to constitute a qualitative account of algebraic equations and functions. So far, four books, in two volumes, have been published, viz., (I) a general introduction, (II) correspondences, (III) the theory of plane curves based on polars, (IV) the singularities of curves. A third volume is planned, in order to deal with the properties of curves invariant under birational transformations; perhaps in this the function-theoretic analysis will receive greater emphasis. Many methods are used for the analysis of the same problem, some for heuristic, some for rigoristic purposes and some for additional insight into the various ramifications of the problem. The various books stand as far as practicable on their own feet and may to that extent be read independently, and many of the "notes" or complements may be omitted in a first reading. Historical notes receive considerable emphasis throughout. What will much delight and instruct the reader is the frequent discussion of paradoxes and sophisms, which clarify the assumptions on which theorems rest.

Algebraic geometry is necessarily of the complex domain, as distinguished from that which seeks merely to illustrate the infinitesimal analysis of real functions, and equally naturally is, as far as achievable, a geometry *in extenso*. The methods which are developed may also be generalized and extended to give information about hyperspaces and algebraic varieties in an arbitrary number of dimensions.

Book I starts with an analysis of groups of points on a line and formations invariant under projective transformations, with special reference to groups of four points, involutions and the equations of the third and fourth degrees. Then the equation $f(x, y) = 0$ is analyzed, both as curve and as correspondence between projective forms of the first grade, and quadratic transformations are introduced to replace "infinitely near" for abstract geometry. The desirability of using the geometry of space of an arbitrary number of dimensions is seen not merely in the possibility of such generalizations as that of Plückerian coordinates to hyperspaces immersed in an n-space, or linear systems as abstract geometries, but also in relation to the essential logical structure of the subject.

Consider in fact a question relating to the correspondence of algebraic varieties, in connection with Netto's theorem (Book I, Ch. III). Suppose that between the algebraic varieties V_n, V_m, themselves irreducible, there exists an irreducible correspondence such that to a generic point of V_n correspond ∞^r points of V_m $(r \geqq 0)$, and a generic point of V_m corresponds to ∞^s points of V_n. We can imagine this correspondence established by a V_q in a V_{n+m},—in the same way as a correspondence between points x and points y on a line is defined by an $f(x, y) = 0$ in S_2,—and the V_q so formed will be irreducible. Netto's theorem, of which the author gives a special proof for algebraic varieties, then gives the result: $q = n + r = m + s$ or $n = m + s - r$. For simple applications of this theorem, for example to the finding of the dimension of the space of lines in S_3, where V_m is S_3 and the correspondence is of line to bundle, there is no difficulty; but in more complicated cases it would seem that unless one took care actually to find the algebraic coordinates of the varieties involved, one's road would be tangled with questions of reducibility, and "sofismi" and "paradossi" would beset it.

Netto's theorem is thus made the basis for the meaning of "in general", e. g., the difference between "in general" for a correspondence $[m, n]$ and a curve of degree $m + n$, the counting of constants, the theory of dimensions, and the theory of compatibility of algebraic equations, leading to the principle of Plücker-Clebsch. The latter principle may be briefly stated in this form: if for generic values of parameters a set of algebraic equations is incompatible, then when it does for particular values become consistent it also becomes indeterminate; more exactly, if a system of $m = n - r$ equations in n unknowns $(r \geqq 0)$ is consistent and admits ∞^r, not ∞^{r+1}, solutions for some particular values of the parameters it is compatible (admitting ∞^r solutions) for generic values of the parameters.

In Book II the emphasis is on correspondences and the "principle of correspondence". The latter deals with the $m + n$ fixed points of an $[m, n]$ correspondence of points on a line. The relations between these and the multiple points are given by Zeuthen's rule. The book contains a general theory of involutions of order higher than two, an elementary theory of plane curves and a chapter on the real representation of the imaginary and Riemann surfaces. Needless to say, the author's philosophical predilections make such general discussions extremely interesting.

The independence of the various books makes it possible, if one so desires, to read the next part, Book III, in large measure before Book II. In fact to many a reader the theory of plane curves based on polars, therein developed, will be the most familiar, although he will perhaps be a little startled by the discussion of one-sided (or odd) and two sided (or even) branches of curves, in the analysis of the real quartic in the projective plane. An algebraic curve without double points cannot have more than one odd branch, and if of even order cannot have any. The section on numerative geometry is extended to twisted curves and hyperspaces.

The last book starts again from elementary considerations and by means of Puiseux developments, quadratic transformations and differential methods, develops the theory of singularities of algebraic curves. The singularities of twisted curves and algebraic varieties are analyzed by means of projections; i. e.,— a curve in S_r is projected on a plane by means of the variable S_{r-2} which joins its generic point with a fixed S_{r-3}. An arbitrary plane curve, with arbitrary singularities, can arise in this way from a twisted curve in hyperspace which has no double points. Similarly, an arbitrary surface in S_3 is transformable into a surface F of a convenient hyperspace ($r \geqq 5$) without singularities, a fact by means of which is established the theorem that the surface can also by a birational transformation be transformed into another surface of S_3 with a double nodal curve having merely triple points (triple at the same time for the curve and for the surface).

For a person whose geometrical background is not more extensive than that of the reviewer some of the proofs may seem to be clouded by ambiguous wording or lack of sufficient detail, and some of the fundamentals, like the theory of quadratic transformations, not to be developed with sufficient broadness. The discussion of the relation of the equation of the fifth degree to the finite groups of projectivities is decidedly incomplete. Yet one has only to compare the book in quantity of material with Salmon, or in point of view and range of subject with almost any book in English on algebraic curves, to realize that it is the work of a rare scholar as well as a notable contributor to mathematics (and epistemology). Writers of minor and major theses are recommended to it.

G. C. EVANS