critique of that defence.* The book under review assists in the perpetuation of the legend, based upon incorrect evidence, that Sir Isaac Newton's delay of twenty years in the announcement of his law of universal gravitation was due to measurements of the size of the earth far below the true value, and that only Picard's determination finally enabled Newton to verify his law. There is very strong evidence in support of the view that Newton's difficulties were of a wholly different character and related to the unsolved problem of the attraction of a spheroid upon an external particle.† In presenting ancient conceptions on the atomic theory and on infinity, C. J. Keyser's article on Lucretius would have afforded illuminating information.‡ Lucretius deserves attention also in the presentation of ancient conceptions of heat.§ But strange to say, Reymond makes only a passing reference to him. In presenting the place of Pliny in the history of science, a reference to the monumental work of Lynn Thorndike|| would have been very much to the point. It is a bit strange that in discussing the ancient abacus and the lunes of Hippocrates of Chios, Ball's *Short History of Mathematics* should be the only authority cited.

The author's style of exposition is clear. Readers will find the book entertaining and, in general, quite accurate.

FLORIAN CAJORI

THE ARITHMETIC OF NICOMACHUS

Nicomachus of Gerasa, *Introduction to Arithmetic*. Translated by Martin Luther D'Ooge, with studies in Greek Arithmetic by Frank Egleston Robbins and Louis Charles Karpinski. New York, Macmillan, 1926. x+318 pp. Price $3.50.

This important work appears as volume XVI of the Humanistic Series of the University of Michigan Studies, a series that is doing much to establish and maintain a high standard of scholarship in this country. The translation was made by the late Professor D'Ooge, whose death eleven years ago was the occasion of a great loss not merely to his university but to the cause of classical scholarship everywhere. It had been completed at the time of his death, but the "supporting studies," as the editors call them, were undertaken later by Professors Robbins and Karpinski. Professor Robbins contributed chapters on the development of Greek arithmetic before Nicomachus; on the latter's life, works, and philosophy; on his philosophy of number, his translators and commentators; on the manuscripts and texts of his works; and on his language and style. Professor Karpinski's contributions consist of chapters on the sources of Greek

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* SCHOOL SCIENCE AND MATHEMATICS, vol. 21 (1921), p. 638 ff.

† W. W. R. Ball, *An Essay on Newton's Principia*, 1893, p. 7; see also the ARCHIVIO DI STORIA DELLA SCIENZA, vol. 3 (1922), pp. 201–204.

‡ C. J. Keyser, this BULLETIN, vol. 24 (1918), p. 321.

§ ISIS, vol. 4 (1922), p. 483–492.

|| *History of Magic and Experimental Science*, vol. 1 (1923), pp. 42–99.

mathematics, on the contents of the *mathematica*, on the Greek numerals, on the successors of Nicomachus (part of the chapter), and on a certain theorem found in the *Introductio*.

It is no disparagement of the scholarship displayed in the "supporting studies" to say that the importance of the work will doubtless be judged to lie largely in the translation itself, the first complete one in our language, and one long needed. The treatise is so important in its bearing upon the history of mathematics and, indeed, upon the history of human thought, that it should long since have been made available to non-classical students. On the other hand, it is a matter of congratulation that the translation was not undertaken by one less worthy of the task, and men are rarely found who are as gifted as Professor D'Ooge in the knowledge of Greek and in the ability to express the delicate shades of that language in equally delicate English.

Nicomachus did for the ancient theory of numbers what Euclid had already done for elementary geometry and Apollonius for conics,—he systematized the knowledge of his predecessors and expressed the result with simplicity of language and with a clearness that appealed to his people. Through Boethius, who embodied in a popular Latin treatise part of the theory thus set forth in Greek, his influence dominated the teaching of the subject for more than fourteen centuries.

The translation itself is, like Jowett's *Plato*, a delightful piece of English, revealing not a little of the power and literary style of the Greek. Only one who was imbued with the spirit of the language could have expressed himself so simply and so directly. The editors do not tell us whether the notes are due to Professor D'Ooge or to themselves, but in either case they are models of what should be expected in a work of this kind, not so much in elucidating the text as in giving valuable information relating to technical terms and to the works of such writers as Euclid, Theon of Smyrna, and Iamblichus.

In the studies the reader will find evidence of a large amount of scholarly research,—to use a term "defamed by every charlatan and soiled with all ignoble use,"—and he will be particularly indebted to the authors for valuable information relating to the life and works of Nicomachus. Naturally, however, he will find a considerable diversity of style, due to the fact that the treatise represents no less than three collaborators whose diction and taste vary to a noticeable degree. He will be quite sure that Professor D'Ooge would not have used both "demonstrative geometry" and "demonstrational geometry" (p. 4) in the same chapter, even with the half-hearted support of Murray, nor the obsolescent spelling "parallelopiped" (p. 57), nor both the forms "Hero" and "Heron," nor both "arithmetic" and "arithmetical" as adjectives. The use of the form *Abbaci* in speaking of the *Liber Abaci* of Fibonacci, simply because the Italian title page gave a form which Leonardo did not use in his *Incipit*, will also strike the reader as foreign to Professor D'Ooge's style and method.

On the other side, readers who are conversant with the history of mathematics will regret a tendency which is frequently and painfully apparent to make sweeping assertions that are not confirmed by the context or by the notes, and which are open to serious doubt. A few of these will serve to illustrate their nature.

P. 4. In the "ἀριθμητική of which the arithmetic of Nicomachus is a specimen, ... we have . . . . the forms of proof and the rigor of the demonstrational (*sic*) geometry." Certainly the forms of proof are entirely different from those of Euclid, and few would place them in the same class for logical rigor.

P. 5. "For the sources of the early Greek arithmetical sciences we must look to Egypt and Babylon, possibly even beyond to India and China." If this means anything definite, it means that traces of the Greek number theory as developed by the Pythagoreans are to be found there, but certainly there is nothing in the text or the notes to justify any such assertion. The mere fact that people could count, could write numbers, and could recognize odd and even numbers at a very early period, as well as use simple problems in series, affords not the slightest historical evidence of any concept of the Greek number theory, as the term is commonly understood.

P. 9. In speaking of "early Egyptian mathematical science" the assertion is made that the "few surviving documents give indications of development along many different lines of mathematical thought." A reader might legitimately look for some information as to the documents referred to, and as to the "many different" topics. There may occur to him a few rhetorical equations, the weak mensuration work of Ahmes, and the probable truncated pyramid in the Moscow papyrus, but what are the "*many different lines*" discussed in the "documents"? The doubt cannot be removed by the other sweeping statement about "further *definite* indications of *real progress* in mathematical *thinking* among the Egyptians." The reader is certainly justified in asking for some evidence that will stand scrutiny, even in the form of note references.

Pp. 10, 11. The statement is made that "the most notable advance in astronomy in Babylon was undoubtedly made during the period in which the science was making real progress in Greece." Since this "real progress" was presumably made in the period from Aristarchus (*c.* 270 B.C.) to Ptolemy (*c.* 150 A.D.), it will be interesting to know the details of the "most notable advance in Babylon," even if this term is taken to mean Babylonia, and Babylonia is taken to mean Chaldea, as is quite proper. Just what "most notable advance" was made after the Persian conquest of 539 B.C., which was the period of Pythagoras; or after the conquest by Alexander, which also antedated Aristarchus? If on the other hand, the "*real* progress" in Greek astronomy was made before Aristarchus, who made it, and what was the date of "the most notable advance in astronomy in Babylon"?

The above are merely typical of questions that arise in connection with various general assertions in the text. A few further notes are given below as typical of certain minor questions that will occur to nearly every reader.

P. 66. The use of the term "geometrical numerals," referring to the early Minoan type.

The validity of the *ex cathedra* statement: "Heath's assertion that we reckon 'with words' is not correct." It might interest a psychologist who was explaining various types of images.

The implied contradiction of Heath's opinion that the Greek notation "did not adversely affect their arithmetic." It would be interesting to see the argument that it really did affect their *arithmetica*.

P. 67. The statement that the short-legged form of Π came after the isosceles form.

P. 68. The use of small modern Greek letters and accents instead of forms more nearly like those found in the early inscriptions and manuscripts.

The use of σαμπῖ as if it were the ancient Greek name for the character for 900.

Pp. 138–140. The indiscriminate use of Isidore and Isidorus, without any apparent reason.

P. 145. The use of San Sepulchri for San Sepolchro or San Sepolcro, or for Santi Sepulchri in case the Latin genitive is given with *Prefatio* as in the 1494 edition. The form San Sepulchri is neither Italian nor Latin.

The spelling Marg*h*arita Philosophica.

It may be allowable to suggest also, that the force of certain passages would have been greater without such expressions as "I hold that" (p. 12), "it seemed to me" (p. 289), and "This gives my theorem" (p. 290). As to this latter interesting relation, it would not have been claimed by the writer of the above statement about "my theorem" if he had looked more fully into the history of the subject. Mr. Jekuthial Ginsburg, who has done considerable work in the history of number theory, has called the reviewer's attention to the fact that Bretschneider not only gave it over eighty years ago (Grunert's ARCHIV, I, 415) but extended it to even powers as well, saying: "Turner's Theorem is a special case of a far-reaching general theorem which I discovered years ago but did not consider new." The name "Turner's Theorem," suggested by Sir William Rowan Hamilton, was ridiculed by Boncompagni, Turner having merely proved the statement made by Nicomachus, as others had done before him. Bretschneider showed, as part of his general theorem, that every odd power $2n+1$ of an integer p is equal to the sum of p^n consecutive odd numbers beginning with $p^n(p-1)+1$. In particular, in the case of seventh powers, as given in the article under review, $n=3$. Then, for example, for 4^7 we have $p=4$, and so the sum of 4^3, or 64, consecutive odd numbers beginning with $4^3(4-1)+1$, or 193, is 4^7. In other words, this well known theorem is precisely the one now claimed as a new discovery,—as, indeed, was very likely the case.

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