with the concluding sentence of the preface: "The field is indeed a most fascinating one, and I believe a course of this character must be given in all our colleges and universities if we are to be fair to our students." The reviewer believes that departments of mathematics, where they have competent instructors, should take the lead in organizing such courses. This book could well furnish a part of the basis for such a course.

R. D. CARMICHAEL

DICKSON ON MODERN ALGEBRA

Modern Algebraic Theories. By L. E. Dickson. Chicago, New York, Boston, Sanborn, 1926. ix+276 pp.

At last English speaking students have a clear, concise guide to the essentials of each of the great branches of modern algebra, written by an authority who has himself enriched several of them by his own outstanding contributions. This publication of Professor Dickson's matured synopsis of modern algebraic theories is a notable event for American mathematics in at least two respects. First, the book will doubtless be for many years the vade mecum of successive generations of graduate students seeking to penetrate the wide and increasingly significant domain of modern algebra. Second, it may suggest to other publishers that it pays to serve mathematics by the publication of something better than inflated advertising matter and mediocre sophomore texts which live their two or three semesters and become liabilities. As it is but seldom that an American publishing house shows sufficient interest in the advancement of science to bring out a work of the calibre of this book, it is to be hoped and expected that the mathematical public will express its appreciation of the publisher's farsighted enterprise in a tangible manner.

Linear algebras, and certain other advanced parts of modern algebra are not discussed in the present treatise; for these the reader is referred to the author's *Algebras and their Arithmetics* (soon to appear in amplified form in a German translation). Presupposing only a knowledge of the rudiments of the calculus and of the elementary theory of equations, *Modern Algebraic Theories* gives rapid, crystal-clear introductions to algebraic invariants, "higher algebra" as usually understood in America, the Galois theory of algebraic equations, the theory of finite linear groups including Klein's theory of the icosahedron and equations of the fifth degree, and group characters.

In the first chapter of only 23 pages the leading concepts of algebraic invariants in the non-symbolic notation are laid down so swiftly that on pp. 17–20 the author obtains the fundamental systems of covariants for the binary p-ic, $p<5$, exhibits the syzygies between them, and on page 23 reaches the solution of the quartic equation from the canonical form of the binary quartic, having defined and proved the covariance of Hessians, Jacobians and discriminants on the way, all with a minimum of computation. Only the essentials are treated; the swarms of insignificant minutiae in which less experienced writers revel, are ignored. The second chapter drives straight ahead, undeflected from the main goal by trivial temptations, until at p. 35 the existence of a fundamental system of covariants of every set of binary forms is proved and on the same page the finiteness of syzygies is established. It is no small achievement to have presented simply and intelligibly so much good mathematics in so little space.

Under the section on "higher algebra" (Chapters III–VI) we find a detailed exposition of linear forms, linear transformations, matrices, quadratic and Hermitian forms, symmetric and Hermitian bilinear forms, invariant factors and elementary divisors, pairs of bilinear, quadratic, and Hermitian forms, all in about 95 pages. The treatment is fresh and in several important respects original with the author. Two novel features may be particularly noted. In Chapter IV the study of two symmetric or two Hermitian bilinear forms is made to depend upon that of two quadratic or two Hermitian forms and the last two problems are treated together by a new, simple device, first introduced in the author's Chicago lectures, so that the theory of the four species of forms, quadratic, Hermitian, symmetric and Hermitian bilinear is unified and presented in but little more space than has hitherto been required for even a concise exposition of any one of the four. Second, in Chapter VI, on pairs of bilinear, quadratic and Hermitian forms, the author presents a new and simpler theory of the equivalence of pairs of such forms. Incidentally exact care is given to all questions of rationality.

This part of higher algebra, long disregarded by mathematical physicists as but another bizarre pastime of the pure mathematicians, having recently become prominent in theories of quanta, has acquired a new significance, and it seems likely that the well trained theoretical physicist may henceforth find it suggestive to accord to algebra a share of the attention which he has lavished on analysis. Matrices, bilinear and Hermitian forms, the highly finished weapons of the pure mathematician, have passed suddenly into the armory of the aggressive questioner of nature. So it has been in the past, times out of number, with abstract theories and so, believe those who give their lives to pure mathematics, will it be in the future.

Naturally the exposition of so extensive a field as that of "higher algebra" in such narrow compass implies close writing and closer reasoning. As everywhere in the book not a word is wasted. On the other hand clarity is not sacrificed to brevity. It is easy to achieve condensation by the Laplacian device of substituting for proof the discouraging falsehood "it is easy to see," but Professor Dickson has not descended to this stale trick of the indolent. No link in the strong chains of reasoning is slurred; the final product is fool-proof. This last does not imply of course that the book is designed for the feeble-minded or the lazy; the best students will find on nearly every page a sentence or two worthy of their sinews. Some of the theories treated—especially in the last half of the book—are inherently difficult for beginners unaccustomed to abstract reasoning; mere expansion will not render the essential nub less hard but will only obscure it in a haze of irrelevancies. Brevity often is the soul of clearness.

The theme in Chapters VII–X is algebraic equations with the Galois theory in the place of honor. Readers approaching this somewhat abstract theory for the first time will find here a remarkably concrete presentation, leading by easy gradations from the familiar cubic and quartic equations through the first principles of substitution groups to Galois resolvents and the group of an equation, with a short intermediary chapter (VIII) on fields, adjunction and reducibility, reaching finally, after only 40 pages from the elementary beginning of the whole story, the group of the general equation of the nth degree. Chapter X, of 25 pages, disposes of the problem of solvability by radicals, and does so in a fashion which should convince anyone capable of human reason. This chapter and the next may be particularly recommended to those (their tribe is far from extinct) who still receive patiently alleged solutions of the general equation of the fifth degree, duplications of the cube and trisections. The just criticisms of Wantzel's revision of Abel's proof that the general equation of degree >4 is not solvable by radicals, and of Abel's own proof, are not evaded here as in certain other works on algebraic equations, but are fairly stated and, what is more important, answered by sound proofs.

The matter in Chapters VII–X is self-contained, and it is probably the shortest, as it is easily one of the clearest, expositions of the modern theory of algebraic equations. These chapters could be read independently of the rest of the book. The converse is not true, however; certain subsequent sections presuppose for their understanding a thorough mastery of these.

The course in Galois theory here presented is reared from first principles with everywhere a minimum of computations, and it is well within the range of able undergraduates. Unlike some phases of modern algebra this theory demands for its comprehension a native capacity for abstract thought rather than technical facility in ingenious manipulations. It is thus one of the infallible touchstones for separating the born mathematicians from the goats. There is no good reason why the ambitious stripling should not test his powers early on the Galois theory instead of deferring acquaintanceship with mathematics as mathematicians like it till the first year of graduate work. To such Professor Dickson's short, well-knit treatise on this great theory may be unreservedly recommended as the adequate means for self-instruction.

In the six pages of Chapter XI the student is discouraged from attempts to trisect all angles, and he is shown exactly what regular polygons can be constructed by the methods of euclidean geometry. Here a judicious use of the previously developed group theory greatly shortens the proofs.

Chapter XII, concerning Tschirnhaus transformations, the Bring-Jerrard normal form, and Brioschi's normal form of quintic equations, is preliminary to XIII, the first part of which presents with remarkable simplicity the essentials of Klein's *Ikosaeder*. Material simplifications due to the author make the treatment particularly fresh and pleasing.

Presupposing a minimum of previous knowledge the author gives (Chapter XIII) in 29 pages a direct derivation of each finite group of linear transformations on two variables, thus acquainting the reader with the outstanding landmarks of Klein's masterpiece so far as they relate to algebra. Here we see algebra at its best, bold of imagination, untrammelled by mean calculations and mighty in the mastery of age-old problems. That so familiar a thing as the icosahedron which Plato doted on almost to the verge of superstition should hold the true key to the riddle of quintic equations, sought for in vain by the keenest algebraists of the eighteenth and early nineteenth centuries, and grasped first in its entirety only by the many-minded Klein after Hermite and Kronecker had done great things with an older magic, must ever remain one of the perfect revelations of pure science. This fascinating chapter XIII includes full discussions of the related form problems and the Galois group of the icosahedral equation; the solution of the general quintic is carried up to the point prescribed by the limits of the book, namely to that of the icosahedral equation, where algebra merges into the theories of special functions. It will be a cold reader indeed who is not inspired by this presentation to proceed the remaining steps of the way and see how the quintic equation is finally solved explicitly in a domain beyond algebra.

Early in Chapter XIV, a sequel to the preceding, the representation of any substitution group as a linear group is established. Inter alia this chapter discusses group matrices, group characters, the computation of the latter and their application to the former. Probably the strongest attraction which this chapter will have for most readers is the easy introduction which it offers to the powerful method of group characters, following (with simplifications) the exposition of Schur, as this efficient tool is none too accessible for beginners in the classic memoirs. As pointed out by the author this theory has unearthed treasures for finite groups not previously known, so that it is not merely an elegant retelling of an old story. It is perhaps as close to an algorithm as any method at present possessed by discrete groups, and it deserves to be mastered by a greater number of students than now use it.

Throughout the book numerous short sets of approachable exercises are provided for the reader to test his mastery of the several theories. These have been judiciously selected and are not mere pockets of hard nuts from crabbed papers for which there was no room in the text. There are also occasional historical notes where the matter is novel or of sufficient interest to justify an allusion, and five extremely helpful lists of references for further reading, much of it in English.

Professor Dickson has given us a gem of mathematical exposition, glowing with the fire of the master, and touched with the creative spark of the living mathematician. This book could not have been written by one not himself an original algebraist. The reader who will diligently understand what he reads will see that algebra is not the wretched hodge podge of shifty tricks which some writers would make it, but a compact science and a high art.

E. T. Bell