A CORRECTION

BY W. L. AYRES

Dr. G. T. Whyburn has kindly called to my attention the fact that Theorems IV and V of my paper, *Concerning the boundaries of domains of a continuous curve*,* are incorrect. The following example constructed by Whyburn shows that theorem IV is false and a somewhat similar example may be used for Theorem V. Let K denote the rectangle with vertices (1, 1), (1, −1), (−1, 1) and (−1, −1) together with its interior. Let C_{-1} and C_0 be the circles with centers (0, −1) and (0, 1) and radii 1 and $\frac{1}{3}$ respectively. For each positive integer n let C_n be the circle with center $(1/2^n, (-1)^n)$ and radius $5/2^{n+3}$. Let C' and C'' be the set of all points (x, y) of $\sum_{i=-1}^{\infty} C_i$ for which $y>1$ and $y<-1$ respectively. Let $M=K+C'+C''$. Let D_{-1} be the set of all points (x, y) of K for which $x<0$ and $y<0$. Let D_0 be the set of all points (x, y) of K for which $x>\frac{1}{2}$. For each positive integer n let D_n be the set of all points (x, y) of K such that $1/2^{2n}>x>1/2^{2n+1}$. Let

$$D = C'' + \sum_{i=-1}^{\infty} D_i$$

and let P be the point (−1, 1). The hypothesis of theorem IV is satisfied but one of the maximal connected subsets of the M-boundary of D with respect to P consists of the interval from (0, 1) to (0, −1) together with the interval from (−1, 0) to (0, 0).

I have found that Theorems IV and V are true if *either* of the following conditions be added to the hypotheses:

(4) *If B_1 is any maximal connected subset of B containing more than one point, then B_1 contains two non-cut-points which are accessible from both D and R.*†

* This Bulletin, vol. 33 (1927), pp. 565–571.

† The maximal connected subset of $M-D$ containing P is denoted by R.

(4′) *If B_1 is any maximal connected subset of B containing more than one point, then the set of non-cut-points of B_1 is accessible from either D or R.*

If condition (4) be added, then these theorems may be proved by practically the same argument that I used. If condition (4′) be added, these theorems may be established by the following argument. The argument follows my "proof" of Theorem IV through line 7 page 569. As x and y are accessible from either D or R (we will suppose D), there is an arc xuy which lies in D except for x and y. Let T be a maximal connected subset of B_1-xzy. Suppose T has a limit point t on $<xzy>$. It is not difficult to see that T contains a non-cut-point s of B_1. There exists an arc st which lies in T except for the point t.* By (4′) there exists an arc rs which lies in D except for s and has only the point r in common with xuy. Now the set $xzy+xuy+rs+st$ divides the plane into three domains, one of which contains R. But if H is the domain containing R there is a point Q of B_1 which belongs neither to H nor to its boundary. Then Q cannot be a limit point of R. Hence every limit point of T belongs to the set $x+y$ and, since neither x nor y is a cut-point of B_1, both x and y are limit points of T. Then there is an arc xvy which lies in T except for x and y. Since every point of $xvy+xzy$ is a limit point of R, R lies in the complementary domain of $xuy+xvy+xzy$ which has $xvy+xzy$ as its boundary. Then the exterior of $xzy+xvy$ contains either D or R and the interior contains the other. Then $B=xzy+xvy$.

THE UNIVERSITY OF TEXAS

* See R. L. Wilder, *Concerning continuous curves*, Fundamenta Mathematicae, vol. 7 (1925), Theorem 1, p. 342.