THE APRIL MEETING OF THE SAN FRANCISCO SECTION

The fifty-fourth regular meeting of the San Francisco Section was held at Stanford University on Saturday, April 7, 1928. In the absence of the regular Chairman, Professor E. R. Hedrick presided during the early part of the meeting and Professor E. W. Brown during the latter part. The total attendance was twenty-seven, including the following twenty members of the Society:

Bernstein, Biggerstaff, Blichfeldt, Bright, E. W. Brown, Buck, Cajori, Crum, E. R. Hedrick, Hoskins, R. L. Jackson, Vern James, D. N. Lehmer, F. R. Morris, Noble, Pehrson, T. M. Putnam, Stager, A. Pell Wheeler, A. R. Williams.

The Secretary announced that the Summer meeting of the Section will be held at Reed College on June 2. It was decided to hold the next Spring meeting at Stanford University on April 6, 1929.

Titles and abstracts of papers read at the meeting follow.

1. Professor E. T. Bell: *Certain class—number relations implicit in the Nachlass of Gauss.*

This paper appears in full in the present issue of this Bulletin.

2. Professor E. T. Bell: *Remark on the number of classes of binary quadratic forms of a given negative determinant.*

The long sought means are indicated for an independent (strictly elementary) proof of the fact that, if p is an odd prime, the number of quadratic residues in the set $1, 2, 3, \cdots, (p-1)/2$ exceeds the number of quadratic non-residues. This paper will be published in the Proceedings of the National Academy.

3. Professor E. T. Bell: *Remarks on the quaternary quadratic identity of Hermite.*

In a recent note in the American Mathematical Monthly, it was observed that Hermite's identity does not admit of generalization by algebraic transformations of the coefficients. The present remarks refer to unpublished work concerning the composition of Hermitian forms in 4 or 8 indeterminates.

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4. Professor E. T. Bell: *Note on difference equations defining enumerative arithmetical functions.*

The trivial algebraic origin of an extensive class of definitive difference equations for enumerative numerical functions is indicated. In particular, the important identity of J. V. Ouspensky (Bulletin de l'Académie des Sciences de l'URSS, 1925, p. 647), concerning representations as sums of squares, is traced to its obvious origin in the formula for the derivative of a product. The note will be published in the Bulletin of the Calcutta Mathematical Society.

5. Professor E. T. Bell: *A generalization of circulants.*

This is a self-contained, independent detail in a recasting of the theory of algebraic numbers in terms of finite processes only. It will appear in the Proceedings of the Edinburgh Mathematical Society.

6. Professor E. T. Bell: *Ternary characteristics of primes.*

This paper has appeared in full in the May-June number of this Bulletin.

7. Professor E. T. Bell: *Certain completely solvable systems of simultaneous diophantine equations.*

This paper will appear shortly in the American Mathematical Monthly. The equations considered are of any degree in any number of indeterminates.

8. Professor E. T. Bell: *A property of resultants.*

The factorization property of the resultant of two algebraic equations, one of which is binomial, is extended to any pair of algebraic equations. The paper will appear in the Messenger of Mathematics.

9. Professor Florian Cajori: *Early history of partial differential equations and of partial differentiation and partial integration.*

This paper is a criticism of recent European expositions of the theory of fluxions, according to which partial fluxional equations are impossible. The history of partial processes and of partial differential equations is traced on the Continent to the time of Euler, and in Great Britain to the opening of the nineteenth century.

10. Professor Florian Cajori: *A revaluation of Harriot's "Artis analyticae praxis."*

John Wallis's appraisement of Harriot's algebra was strained by national bias. Conflicting estimates of later date, and unwarranted statements found in the most recent histories, induced the present author to make a re-examination of Harriot's book.

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11. Professor E. R. Hedrick: *On derivatives of non-analytic functions.*

Kasner has recently shown (Proceedings of the National Academy, vol. 13 (1928), pp. 75–82) that the increment ratio $\Delta w/\Delta z$ for a non-analytic function $w=f(z)$ approaches points on the circumference of a circle as the increments approach zero. The present paper emphasizes the fact that one diameter of the Kasner circle is determined by $\partial w/\partial x$ and $\partial w/\partial(yi)$, where $z=x+yi$, and that another diameter is determined by the maximum and minimum values of $r=|\partial w/\partial z|^2$, which was discussed by Hedrick, Ingold, and Westfall (Journal de Mathematiques, vol. 2 (1923), pp. 327–342). The Tissot indicatrix modified in a manner that is not essential, discussed in that paper is in fact the polar diagram of the distances from the origin to the points of the Kasner circle. The diameter of the Kasner circle is the *ellipticity* discussed in that paper, when a nonessential modification is made. The fact that the Tissot characteristic lines are orthogonal corresponds to Kasner's theorem that orthogonal directions give points at the ends of a diameter of the Kasner circle. This paper will appear in the Proceedings of the National Academy.

12. Professor E. R. Hedrick: *On the increment-ratio for non-analytic functions.*

The increment-ratio $q=\Delta w/\Delta z=[f(z)-f(a)]/(z-a)$ for a function $w=f(z)$ may be considered as a new function of z. Its properties are partly known through those of its limit, dw/dz, which ordinarily depends upon the slope of the curve on which Δz approaches zero. But $q=r+it$ is defined for all values of z for which $f(z)$ is defined, except perhaps $z=a$. In this paper it is shown by direct calculation that the Jacobian of the transformation from (x, y) to (r, t) vanishes precisely on the Kasner circle. In general, the vanishing of the Jacobian of any non-analytic function defines a curve of which a special case is the branch point of an analytic function; this curve may be called the *edge of regression* on account of its similarity to the edge of regression of a developable surface. The edge of regression of the increment-ratio q is then the Kasner circle, which is the envelope of curves corresponding to any pencil of curves through the point $z=a$ in the z plane. The Riemann surface over the q plane is, in general, two-leaved, joined along the Kasner circle. However, not every function whose edge of regression is a circle can be set up as an increment-ratio. The result of this paper will appear in the Proceedings of the National Academy; a detailed proof will appear in the Commemoration Volume (1928) of the Calcutta Mathematical Society.

13. Professor E. R. Hedrick: *Analytic points of non-analytic functions.*

The derivative of a non-analytic function $w=f(z)$, $w=u+vi$, $z=x+yi$ may exist at a point $z=a$, and it does if $u_x=v_y$, $u_y=-v_x$. Such a point will be called an *analytic point.* If the Jacobian, J, of u and v does not

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vanish, $f(z)$ near a is given by a finite Taylor expansion in the powers of $(z-a)$, plus a remainder, $(z-a)^n Q_n(z)$, where $Q_n(z)$ is not analytic at $z=a$. The analytic point can be "*removed*" by subtracting from $f(z)$ a polynomial and dividing by $(z-a)^n$. Any finite number of analytic points can be removed. Under obvious conditions, the expansion is infinite and $f(z)$ is analytic in a region. A limit point l of such analytic points is itself an analytic point. The functions $Q_n(z)$ for $z=l$ is non-analytic at $z=l$; hence it is non-analytic except at a finite number of points near l. Then $f(z)$ must have the same property. It follows, under suitable hypotheses, that if $f(z)$ has more than a finite number of analytic points in a closed region, it is analytic in that region. This result generalizes considerably the usual theorems on the independence of the equations $u_x = v_y$, $u_y = -v_x$ when $J \neq 0$.

14. Professor E. R. Hedrick: *On the law of the excluded middle from the axiomatic standpoint.*

It has been assumed by recent writers that the law of the excluded middle would be sustained by those who treat mathematics axiomatically, as well as by believers in a priori validity of the law. In the present paper, it is pointed out that the words "true" and "false" should be defined in an axiomatic discussion, and that the statements "p is false," "p is not true (or untrue)," and "not-p is true" may have different meanings. The following definitions are suggested. Given a set of axioms, a proposition p is called *true* if it can be reached by a finite number of syllogisms. A *contradiction* arises if q is *true* and not-q is *true*. Finally, p is called *false* if the axioms, together with p, lead to a contradiction after a finite number of syllogisms. A set of axioms that lead to a contradiction is called *inconsistent*. It follows as a theorem that if p is *false* and p is *true*, then the axioms are inconsistent: such sets of axioms are possible, though not desirable; to exclude them is to assert the law of contradiction. With such definitions, even if the axioms are consistent, the law of the excluded middle would be open to question, and could not be said to have a priori validity.

15. Dr. A. R. Williams: *Quintic surfaces with two coincident double lines.*

The purpose of this paper is to contrast two kinds of quintic surfaces. Both have a double curve consisting of two coincident lines. But in the first case the lines have approached coincidence remaining in the same plane, and the resulting surface is not, in general, rational. In the second case the lines, in approaching coincidence, have remained skew, and the surface is rational. Five conditions must be added to the general surface of the first type to make it of the second type.

16. Professor E. W. Brown: *Harmonic analysis of the disturbing function, with a remainder formula.*

The author shows how an approximate value for the remainder after some definite term in the expansion of the disturbing function could be

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obtained, and he gives the general series from which this value can be deduced. With its use, harmonic analysis with two arguments would give the coefficients of the needed periodic terms without the extensive numerical calculations which are usually necessary.

B. A. BERNSTEIN,
Secretary of the Section.

A CORRECTION

BY W. J. TRJITZINSKY

My attention has been drawn to the fact that Theorem I of my paper, *Expansion in series of non-inverted factorials*, (this Bulletin, vol. 34 (1928), pp. 193-196), is not new. This theorem is a special case of a theorem found on page 229 of N. E. Nörlund's *Differenzenrechnung*, and is due to Nörlund; it appeared first in Annales de l'Ecole Normale Supérieure ((3), vol. 39 (1922)). The theorem is also a special case of a theorem less general than that of Nörlund, which was proved by Carlson in Nova Acta Soc. Scient. Upsaliensis ((4), vol. 4 (1915)). Neither of these was known to me at the time of the publication of my paper.

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