# TOWNSEND ON REAL VARIABLES

*Functions of Real Variables.* By E. J. Townsend. New York, Henry Holt, 1928. 405 pp. $5.00.

This book, as indicated in the preface, is a text-book on the subject, and not a treatise. The material used has been divided into seven chapters of approximately equal length, except for the first. They are as follows: Chapter I, *Real Number System,*—a short discussion of real numbers as defined by Cantor and Dedekind; Chapter II, *Theory of Point Sets,*—the usual properties involved in the notions of open and closed sets and measure, chiefly confined to linear sets; Chapter III, *Continuity and Discontinuity of Functions,*—including double and iterated limits, continuity in one or two variables, semi-continuous and pointwise discontinuous functions; Chapter IV, *Derivatives and their Properties,*—derivatives, upper and lower derivatives, law of the mean, partial derivatives, and total differentiability; Chapter V, *Riemann Theory of Integration,*—the usual theory, including improper integrals and integrals involving a parameter; Chapter VI, *Lebesgue and Other Integrals,*—the usual definitions and theory, including primitive functions and a short account of the work of Stieltjes' Hellinger, Denjoy, and Perron; Chapter VII, *Infinite Series,*—uniform and quasi-uniform convergence of series of functions, with applications to differentiation and integration, properties of power series (not including Taylor's series), condensation of singularities, and various methods of handling divergent series. Mention should also be made of the collections of problems at the ends of the chapters and the numerous references to original sources, both of which should be useful to teacher and student.

In criticizing this book it would be natural to compare it with the other two books on the same subject in English, those by Hobson and by Pierpont, but it must be remembered that they are essentially treatises, while the book under consideration is a comparatively short text-book and must be judged accordingly. A discussion of its merits falls naturally under three heads: its aesthetic qualities, the choice of material, and its utility.

With regard to the first the book is excellent. Not only is the author's style good, but he has a manner of presentation that makes the book intensely interesting. The publishers, also, are to be congratulated on their workmanship, as the arrangement and printing are artistic and there is a noticeable freedom from misprints.

The choice of material appeals to the writer in general, but not entirely so. The chief sin of omission is the confining of the point-set theory to linear sets and the restriction of so many of the theorems on functions to functions of one variable. To be sure, most of the generalization is immediate, but in some cases this is not true and the beginner cannot be expected to know when this is the case. It also seems strange to see no discussion of Taylor's series and the conditions for its validity in a chapter devoted to infinite series, especially when nearly eight pages are given to power series. On the other hand it is the writer's opinion that the author has to some extent yielded to the temptation to make his book encyclopedic in character and would have done better to omit certain topics so as to permit a thorough treatment of the topics mentioned above.

One who expresses an opinion as to the utility of this book will be largely influenced by what he conceives to be the primary aim of a first course in this subject, whether the imparting of a large body of new information or training in rigorous mathematical reasoning. The writer regards this book as superior in regard to the first of these objectives, but weak in regard to the second. The chief trouble is a willingness to allow the student to accept as true without proof facts which appear to be obvious, a failing most difficult to overcome in the average graduate student. For instance, in the definition of interior measure on p. 65 there is no mention of the necessity of proving uniqueness. The casual statements in various places (for example, p. 277) that the work can be readily extended to $n$ dimensions is an encouragement to loose thinking. The paucity of references to previous sections in demonstrations, although improving the style, adds to the difficulty in regard to this matter, since the student is apt to accept the statements made without verifying them. Furthermore, the author himself is not impeccable in some of his statements. We learn on p. 44 that the sum of any number of closed sets is a closed set, which is of course not true unless a finite number is meant. But on pp. 51, 52 we find a set $E$ which is defined as the sum of an enumerable system of perfect sets $E_n$ and read that "since $E_n$ is a perfect set for each value of $n$, it follows that $E$ is also a perfect set." It is regrettable that a book otherwise so fine should be so marred by this sort of thing.

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## SECOND EDITION OF NETTO'S KOMBINATORIK

*Lehrbuch der Kombinatorik*. By Eugen Netto. Second edition, with notes and additional chapters by Viggo Brun and Th. Skolem. Leipzig and Berlin, B. G. Teubner, 1927. viii+341 pp.

This work is volume 7 of the series B. G. Teubners Sammlung von Lehrbüchern aus dem Gebiete der mathematischen Wissenschaften mit Einschluss ihrer Anwendungen, in which a number of contributors to the Encyclopædie expand their articles in textbook form.

The first edition appeared in 1901 and contained thirteen chapters, which are reprinted without change, pp. 1–258. Th. Skolem contributes notes on these chapters (pp. 309–338) containing simplifications of a number of proofs, as well as extensions of several of the topics treated by Netto. Since no review of the first edition appeared in this Bulletin, it seems appropriate to indicate briefly the contents of the first thirteen chapters. Chapters 1 and 2 give the elementary notions on permutations and combinations, and their connection with the binomial and multinomial theorems. Chapter 3 deals with arrangements of a number of elements with restrictions on the places which the elements may occupy; topics such as the problem of eight queens and Latin squares are treated here. Chapter 4 considers the inversions and sequences in permutations, and Chapter 5