yielded to the temptation to make his book encyclopedic in character and would have done better to omit certain topics so as to permit a thorough treatment of the topics mentioned above.

One who expresses an opinion as to the utility of this book will be largely influenced by what he conceives to be the primary aim of a first course in this subject, whether the imparting of a large body of new information or training in rigorous mathematical reasoning. The writer regards this book as superior in regard to the first of these objectives, but weak in regard to the second. The chief trouble is a willingness to allow the student to accept as true without proof facts which appear to be obvious, a failing most difficult to overcome in the average graduate student. For instance, in the definition of interior measure on p. 65 there is no mention of the necessity of proving uniqueness. The casual statements in various places (for example, p. 277) that the work can be readily extended to n dimensions is an encouragement to loose thinking. The paucity of references to previous sections in demonstrations, although improving the style, adds to the difficulty in regard to this matter, since the student is apt to accept the statements made without verifying them. Furthermore, the author himself is not impeccable in some of his statements. We learn on p. 44 that the sum of any number of closed sets is a closed set, which is of course not true unless a finite number is meant. But on pp. 51, 52 we find a set E which is defined as the sum of an enumerable system of perfect sets E_n and read that "since E_n is a perfect set for each value of n, it follows that E is also a perfect set." It is regrettable that a book otherwise so fine should be so marred by this sort of thing.

<div style="text-align:right">W. A. WILSON</div>

SECOND EDITION OF NETTO'S KOMBINATORIK

Lehrbuch der Kombinatorik. By Eugen Netto. Second edition, with notes and additional chapters by Viggo Brun and Th. Skolem. Leipzig and Berlin, B. G. Teubner, 1927. viii+341 pp.

This work is volume 7 of the series B. G. Teubners Sammlung von Lehrbüchern aus dem Gebiete der mathematischen Wissenschaften mit Einschluss ihrer Anwendungen, in which a number of contributors to the Encyclopædie expand their articles in textbook form.

The first edition appeared in 1901 and contained thirteen chapters, which are reprinted without change, pp. 1–258. Th. Skolem contributes notes on these chapters (pp. 309–338) containing simplifications of a number of proofs, as well as extensions of several of the topics treated by Netto. Since no review of the first edition appeared in this Bulletin, it seems appropriate to indicate briefly the contents of the first thirteen chapters. Chapters 1 and 2 give the elementary notions on permutations and combinations, and their connection with the binomial and multinomial theorems. Chapter 3 deals with arrangements of a number of elements with restrictions on the places which the elements may occupy; topics such as the problem of eight queens and Latin squares are treated here. Chapter 4 considers the inversions and sequences in permutations, and Chapter 5

gives an elementary exposition of various partition problems. The analytic treatment of partitions by means of generating functions follows in Chapter 6, and Chapter 7 contains some applications of partitions to problems in analysis. Chapter 8 considers the combinatorial arrangements as sums of symbolic products, and Chapter 9 is concerned with some special arrangements other than permutations and combinations. Chapter 10 deals with triads, and although Netto quotes some early work of E. H. Moore, Skolem's notes on this chapter contain no reference to the researches of Cole, White and their students. In Chapter 11, the Kirkman problem is considered, and Chapter 12 gives applications of combinatorial analysis to probability and questions in formal algebra such as the inversion of a power series. Chapter 13 contains a collection of formulas, mainly identical relations between binomial coefficients. The reviewer has found this collection very useful on a number of occasions. The new subject matter of the second edition begins with Chapter 14 (by Viggo Brun). Here a very general combinatorial concept is introduced, that of the distribution function

$$D(s_0 \cdots s, \ r_0 \cdots r, \ \cdots, \ w_0 \cdots w; \ \sigma_0 \cdots \sigma, \ \rho_0 \cdots \rho, \ \omega_0 \cdots \omega)$$

which is defined as the number of distributions of from s_0 to s black balls, from r_0 to r red balls, $\cdots$, from w_0 to w white balls in a number of urns of which the first is to contain from σ_0 to σ balls, the second from ρ_0 to ρ balls, $\cdots$, and the last, from ω_0 to ω balls. First, a duality theorem is noted, which says that in D the sets of Latin and Greek letters may be interchanged without changing the value of the distribution function. The recurrent formulas and explicit expressions for D are given, and the results are applied to an extension of the work of Möbius on the factorization of integers treated in Chapter 8 (where the Möbius number-theoretic function $\mu(n)$ is introduced). Reference is also made to Brun's own work on the sieve of Eratosthenes. In Chapter 15, Th. Skolem deals with groupments, combinatorial reciprocities, and systems of pairs. A groupment is a distribution in the sense of the preceding chapter with the restriction that no urn shall contain two balls of the same color. The notion of duality is introduced and generalized, and it is shown that the groupments of systems of pairs have important connections with the theory of triads, as well as with the Petersen graphs in analysis situs. Lack of space prevents the reviewer from giving any details of these somewhat complicated things, but it should be stated that both these last chapters form important contributions to combinatorial analysis, and that they may be read without knowledge of the preceding chapters.

T. H. GRONWALL