But there is an intellectual criterion which is ultimate, which differs from those others which seek the value of a theory outside the theory itself. It is the judgment of its esthetic worth, the appreciation of its structure. It is perhaps more noteworthy that dynamical theory should have acquired a value of this sort than, for example, projective geometry, or the theory of groups. And to our author more perhaps than to any other man belongs the credit that this is so.

<div align="right">B. O. KOOPMAN</div>

THE RHIND PAPYRUS

The Rhind Mathematical Papyrus, British Museum 10057 and 10058, in two
   volumes. Volume I. By Arnold Buffum Chace, with the assistance of
   Henry Parker Manning, and with a bibliography of Egyptian mathematics
   by Raymond Clare Archibald. [x]+210 pp., 1927. Royal 8vo. Volume II.
   By Arnold Buffum Chace, Ludlow Bull, and Henry Parker Manning, with a
   bibliography of Egyptian and Babylonian mathematics (supplement) by
   Raymond Clare Archibald, and a description of the mathematical leather
   roll in the British Museum, by S. R. K. Glanville. Mathematical Associa-
   tion of America, Oberlin, Ohio; xvi pp.+31 photographic plates +109 fac-
   simile plates+109 facing pages of text, 12 pp. of bibliography +8 pp.,
   1929. Royal oblong folio. Price, $20.

The publication of this treatise, the product of nearly twenty years of scholarly work, is an event of such importance in connection with the history of mathematics as to require more than a cursory examination or a brief description. The Rhind (Ahmes, A'h-mosè) Papyrus is the most extensive mathematical treatise written before the sixteenth century B. C. that has come down to us. We have no contemporary manuscripts of any of the Greek classics on geometry, the theory of numbers, or computation. Our knowledge of the Sumerian, Assyrian, Babylonian, and Chaldean mathematics is derived solely from numerical tables, a few tablets containing a little work in mensuration, numerous others relating to commercial life, and some recently studied ones relating to the Pythagorean triangle, the angle inscribed in a semi-circle, and the rule for solving the quadratic. Such Chinese and Hindu sources as we have, relating to the pre-Christian period, are of uncertain authenticity, especially those purporting to be copies of Chinese documents preceding the eleventh century B. C. In the case of Egypt, however, we have, in fairly complete form, the original document written by A'h-mosè (Ahmes) in the reign of 'A-user-Rê' (c. 1650 B. C.), being a copy or a paraphrase of one dating from the reign of Ne-ma'et-Rê' (Amen-em-hât III), 1849–1801 B. C., or at least similar to it. That such a document, written more than a thousand years before mathematics began to make any noteworthy advance in Greek territory, should have come down to us almost intact, is one of the most remarkable incidents connected with source material of any kind. It is also interesting to know that another manuscript, even earlier than this, is soon to be published, the Golenishchev papyrus now in Moscow (*Quellen und Studien zur Geschichte der Mathematik, Abteilung A: Quellen*, Berlin, 1930),

referred to in the bibliography by Professor Archibald. This is probably about 200 years older than the Rhind papyrus, and seems to have been copied from a work going back to about 2000 B. C. It has been known for a number of years, but this publication is the only complete edition of the manuscript.

The A'h-mosè manuscript has long been familiar to historians. In 1877, Eisenlohr published in Leipzig *Ein mathematisches Handbuch der alten Aegypter (Papyrus Rhind des British Museum)*; in 1898 the British Museum published a so-called facsimile, but it was merely from a copy made by hand and contained several errors or omissions; and in 1923 Professor T. E. Peet published a transcription, translation, and commentary which was the first really scientific study of the original manuscript to appear, although the world is deeply indebted to Eisenlohr for his pioneer work. Besides these editions, numerous articles have appeared, all of which are listed in Professor Archibald's bibliography mentioned below.

With respect to the present edition by Dr. Chace and his collaborators, it is the purpose of this review to speak of (1) the general nature of the work, (2) certain of the original contributions to the interpretation of the text, (3) the bibliographical material, (4) the article by Mr. Glanville, (5) the authors and their special contributions, and (6) the mechanical features of the book.

As to the general nature of the manuscript, this is sufficiently well known to all who are familiar with any of the modern histories of mathematics, and hence it is unnecessary to dwell upon it at this time. A glance at the bibliography will show how numerous are the monographs and books in which it has been described.

What Dr. Chace has done, it being understood that the work of his collaborators is also included in such references, is first to give, in volume I, a free and readable translation of the entire manuscript, preceded by a discussion of Egyptian arithmetic, measures, and geometry; a study of the methods and aims of the Egyptian mathematician; a note on the Egyptian calendar; and a statement relating to certain technical terms. This translation is designed to meet the needs of the reader who is interested chiefly in the text and the methods of solving the problems. The mathematical features having been considered in the first volume, the second volume is devoted chiefly to the philological problem, and it is here that the painstaking care of the authors shows itself even more clearly, this being evident even to one who knows little or nothing of the Egyptian language and of the paleographical difficulties which the translators encountered.

This volume contains (1) the only true facsimile of the manuscript ever published, being a photographic reproduction of the entire work; (2) a copy of this in the original two-color hieratic form; (3) a hieroglyphic transcription underneath the hieratic, the latter being the more difficult to read, just as the ordinary handwriting of most people is more difficult than print; (4) a transliteration of this hieroglyphic into Latin letters, arranged from right to left as in the original; and (5) on the facing page a rearrangement of this translation from left to right, with English translation, word for word, underneath. In addition to this there are various explanatory and philological notes to aid the reader, not merely the tyro in such matters but the scholar as well. This arrangement enables the student of Egyptian history, science, and litera-

ture to pass from the hieratic through the various steps which the scholar must take to reach the verbatim translation into English, after which he may turn to the free translation in volume I. It may safely be said that no scientific manuscript of this size and importance has ever before been presented with such a degree of thoroughness as is here shown. Another feature of interest to every reader, and of great value to the layman in interpreting the text, is the introduction to the second volume. In this is explained the general nature of hieratic writing and of the hieroglyphs, and there are set forth a number of important facts relating to grammar, the method of writing fractions (a particularly important subject in this manuscript), and a brief discussion of translation and pronunciation. All this work makes use of the studies of such philologists as Peet, Gardiner, Erman and Grapow, and Griffith, and therefore represents the latest linguistic researches.

As to the photographic reproduction of the original, mention should be made of the small fragments which are now in the New York Historical Society. Professor Peet, working from drawings, succeeded in placing in the text 24 of these fragments, and in the present edition a dozen more have been so placed. Certain of the bits of papyrus preserved with these fragments seem, however, not to have belonged to this manuscript, and are not inserted.

In the hieroglyphic transcription the present edition has taken advantage of the work of Peet and of Gunn's noteworthy review of the latter's treatise, as is shown in numerous footnotes. The authors were also fortunate in having Gardiner's recent Egyptian Grammar (1927) to assist them in their preparation of volume II. Gunn's review was likewise of great help in the translation and explanation of several words not theretofore clearly understood.

With respect to the original contributions to the interpretation of the text it must suffice if a few references are given showing the departure made from the interpretations of Peet and Gunn, and calling attention to an important solution not heretofore made by any writers. Any more extensive list or attempt at explanation would carry this review beyond the limits of space which are allowed. In the following illustrations the references are by volume and page (or problem): I, 5-6, the theory of Egyptian division is treated in a manner that is both original and convincing; I, 7–10, the idea of the common fraction contradicts that of Peet (15–20); I, 4, the concept of $\frac{2}{3}$ as an independent fraction, instead (as Gunn states) being arrived at "via $\frac{1}{3}$"; I, 13, the reason for the table of the division of 2 by odd numbers (compare Peet, p. 34); I, 14, the question of whether the reckoning is a solution (as here stated) or a proof (as in Peet, 34 and as Hultsch some eighteen years earlier asserted); new readings in problems 4 and 5 (in which Gunn suggests an error in the MS); problem 9, the discovery that the unit figures are not missing, as shown by the photograph; I, 36–37, on the question of the area of an isosceles triangle,—the use of the altitude or the length of a side. Dr. Chace takes the latter factor, a position which he himself admits is open to doubt, and which Gunn and Peet (Journal of Egyptian Archaeology, vol. 14, pp., 167–184) have recently shown to be untenable; II, prob. 86, in which the proper arrangement of this problem is shown for the first time. One of the most interesting differences is seen in problem 53. Of this, Professor Peet (p. 95) remarks. "It is hardly worth while to spend much time on a problem which is clearly incomplete and in-

correct. . . . The first calculation is hopeless. . . ." While granting the dif-
ficulty and inaccuracy, Dr. Chace gives (I, 94) a very reasonable explanation
and one which seems worthy to meet with general acceptance. The solution
of problem 40, the rule for a geometric progression (I, 30), the explanation of
the way in which the Egyptians derived their value of π, and the explanation
of problems 28 and 29 (I, 70) seem also to be new. It may not be out of place
in this connection, also, to mention the fact that, in spite of Professor Peet's
scholarly treatment of the subject, some twenty-five errors in his work have
been corrected, besides an umber of minor slips of the pen.

As to the bibliographical material, it is as difficult to speak moderately
as it is to speak in this way of the work as a whole, and particularly of the re-
markable care taken in volume II. Professor Archibald has listed more than
250 books, monographs, and articles on the subject of Egyptian mathematics,
particularly with reference to the Rhind Papyrus, with certain related works
on the contributions of the Babylonians. Not only did he personally examine
each of these, with the exception of a single one now in press (and of which he
had access to a description), but in every important case he has given a brief
statement of the nature and value of the contribution. It is probable that no
similar bibliography of this extent has ever been prepared with the same ac-
curacy and care. The value of the rest of the work is greatly enhanced by its
presence and the inspiration to other bibliographers will be salutary. Since
the book was printed, two other articles of value have appeared,—Gunn and
Peet on four problems in the Moscow papyrus (Journal of Egyptian Archae-
ology for November, 1929), and Vogel on the Rhind papyrus (Munich, 1929).
A supplementary leaflet relating to these has been inserted at the end of every
published copy. In the former article it is suggested that the word used for
cubit is probably the name for arm, and the editors of the Chace edition have
evidence that it means the fullness of the arm,—that is, from the elbow to the
finger-tips.

Mr. Glanville's article on the leather roll, which he first described in 1927,
is also welcome. The document is a contemporary of the Rhind papyrus and
contains little of value not found in the latter. It is here reproduced photo-
graphically, with descriptive matter.

A word should be said as to the authors of such a remarkable treatise and
the knowledge which they have brought to the work. Dr. Chace, Chancellor
(Chairman of the Board of Trustees) of Brown University, was a student at the
University of Paris more than half a century ago. Circumstances took him
into manufacturing lines but never destroyed his taste for scholarly pursuits.
He and Mrs. Chace, some twenty years ago, on a visit to Egypt, determined to
translate the Rhind papyrus. They took up the study of hieroglyphs and
the hieratic writing, and finally, after years of labor, achieved their purpose.
When the time came for considering the final preparation of the text, Dr.
Bull, associate curator of Egyptian antiquities in the Metropolitan Museum
of New York, and Professor Manning of Brown University, each familiar
with both hieroglyphic and hieratic writing, were called to join with Dr. Chace
in the work, particularly with respect to the arduous task of preparing volume
II. As a result of the labors of all concerned, the work as a whole easily
ranks as one of the best efforts yet made to render an early Egyptian work

available for the use of historians and others who may be interested in the subject.

As to Professor Archibald, he stands easily at the head of American bibliographers in the field of mathematics. If any evidence of this fact were needed, the bibliography here published affords ample proof of the assertion. Nor should I fail to mention the fact, known more fully to this reviewer than to most others, that much of the success of the publication is due to the arduous labors of Professor Archibald as the more active of the two members of the Committee on Publication appointed by the Mathematical Association of America. It is due to Dr. Chace to mention one other fact, that he met all the expenses of the publication, and that the income from the sales will all go to the endowment fund of this Association. And finally, in the way of personal matters, a large amount of credit is due to the skillful management of a man who delights chiefly in obscuring his own good works,—Professor H. E. Slaught, of the University of Chicago, who may properly be called the founder of the Association and the chief promotor of all its enterprises.

With respect to the mechanical features, the two volumes stand in a class by themselves. Only a short time ago Mr. William Dana Orcutt published a work entitled, *In Quest of the Perfect Book* (Boston, 1926). It would not be far from the truth if it were said that this product of the Plandome Press is as near a perfect book of its kind, with respect to the mechanical as well as the intellectual features, as we can expect to see for many years to come.

This review is manifestly one of praise, and this is justified. If there were enough in the way of adverse criticism to make its mention of any help to the reader, this would be included. After all, the purpose of a review is to state fairly the general nature of a book and not to hesitate to speak well of a good production any more than to call attention to the defects of a work in which these are of any moment.

DAVID EUGENE SMITH