Leçons sur les Nombres Transfinis. By W. Sierpinski. Paris, Gauthier-Villars, 1928. 240 pp.

The remarkable grace and ease with which this admittedly baffling subject is presented is sure to win for this book a warm welcome in many fields of mathematics. The book appears under the famous Borel collection of monographs, and habitual readers may be well assured that it is quite up to the standard set by previous contributors to this series. No previous knowledge of the subject treated or of the theory of sets is presupposed, and any mathematician who cares to familiarize himself with this field, which has assumed such large proportions in recent years because of its applications in the theory of functions and in the theory of sets, will find here this necessary material in a surprisingly accessible and readable form.

As is to be expected, much is found concerning the modern controversy between the so-called idealists and realists in mathematics, and the existence of that controversy pervades almost every page. The author himself, although lauded in the preface by Borel (who is a realist) as an idealist, which he undoubtedly is, nevertheless presents some of the arguments of the realists and takes pains to establish as much of the treatment as possible on a basis which would be acceptable even to the realists. For example, much stress—and in the reviewer's opinion, far too much stress—is placed on the fact that certain sets are *effectively* countable. The property of a given set M of being effectively countable, that is, of some person's knowing a law by means of which the elements of M may be enumerated, seems to the reviewer to be far too dependent on the human beings living at a given moment to be in any sense a mathematical property of a set. A given set M may be effectively countable tomorrow which is not so today; or worse yet, M may be effectively countable today and cease to be so tomorrow, for conceivably some person might discover a law by means of which the elements of M may be enumerated, and unfortunately die the next day leaving no record of his discovery. I do not like to think of such an elusive property as a mathematical property at all. A mathematical property should be something which is, once for all, either possessed or not possessed by a given set M, and whether or not M possesses this property certainly is independent of the particular group of mathematicians which happen to be alive at a given time. It should be well understood, however, that the reviewer's criticism is directed more at the realist than at the author of the book under review.

As indicated in the title and preface, this treatment is devoted entirely to a study of transfinite numbers in themselves, to the complete exclusion of their numerous applications in other branches of mathematics. It is divided quite naturally into two parts of approximately equal length, the first being devoted to cardinal numbers and the second to ordinal ones. In each part the first five chapters are concerned largely with propositions which can be established without the aid of the axiom of choice or Zermelo Postulate, and in the sixth and last chapter there are given theorems which can be proved only with the aid of this postulate, together with a discussion of some of the alternatives to the postulate and of some of the outstanding unsolved problems in the theory. In each case the arithmetic of and laws of operation with the respective classes of numbers are developed so far as is known, and indeed this is quite surprisingly far. Of particular interest in the first part is a very extensive chapter on countable sets and a slightly shorter one on sets having the power of the continuum.

This treatment is quite satisfactory from the standpoint both of rigor and of clarity. Definitions are, for the most part, clearly and definitely given where needed; only in a very few cases—notably page 142 where the notion of the inverse of an order type is used—could a fuller explanation be desired. The book is not marred, as is too often the case, by a complicated and extensive system of symbolic notation, which contributes much to its readability. The author and the publishers are indeed to be complimented upon the appearance of such a well prepared treatise on such a well selected subject.

G. T. WHYBURN

Modern Geometry. By Roger A. Johnson. Houghton Mifflin, 1929. xiii+319 pp.

The recent appearance of this book on Modern Geometry* will be welcomed by all mathematicians who believe in the intrinsic value of a more general interest in this subject. The content of the book deals with the geometry of the triangle and the circle developed by the elementary concepts of euclidean geometry extended to include some trigonometric functions and circular inversion. The latter topic, which is defined by means of similar figures and proportions, is introduced in an early chapter and used with excellent effect throughout the text. An important feature is the generalization of proofs by the use of "directed angles" as well as "directed distances." By a sane introduction and use of the phrase "points at infinity" one of the transitions to the subject of projective geometry has been simplified for the student.

It seems to the reviewer that this book will be found a valuable text for undergraduates and prospective teachers. At first sight an adequate number of exercises may appear to be lacking for such a purpose, but there are literally hundreds of stated theorems in which the construction and a part or all of the proof are left as exercises and this gives the student a better insight in the progressive development of the subject. The book will be equally valuable as a reference. As the editor has stated "The content of this book, in spite of its elementary character, is by no means well known to mathematicians in general." A vast amount of material has been collected from numerous sources and for this reason the book furnishes a background of the subject and a wealth of information which will be found invaluable in the teaching of elementary geometry.

Doubtless many teachers would have preferred a different arrangement for the topics which are included. Suffice it to say that the author has achieved to a marked degree his purpose as stated in the preface "the unity and harmony of the arrangement and the interrelation of the various parts to one another." We commend this book with the belief that it merits success.

J. I. TRACEY

* For a detailed statement of contents see The American Mathematical Monthly, vol. 36, (November, 1929), p. 482.