SHORTER NOTICES

Einführung in die Algebra. By Otto Haupt. Vols. I–II. Leipzig, Akademische Verlagsgesellschaft, 1929. 663 pp.

The domain of algebra has in recent years been enriched with a great number of German textbooks of high quality. I need only mention the books by Perron, Fricke, Hasse, and Bieberbach's edition of Bauer's algebra. The new algebra by Haupt might therefore at first glance seem somewhat superfluous. A closer study, however, reveals that it as an unusually successful attempt to present the foundations of modern abstract (commutative) algebra within the scope of a textbook.

The book is mainly based on the fundamental ideas of Steinitz set forth in his paper *Algebraische Theorie der Körper* (Journal für Mathematik, vol. 137 (1910), pp. 167–309). It contains numerous references to newer literature and several investigations are here for the first time presented in a textbook. The lucidity of its style makes it a convenient and attractive introduction to some very important parts of modern algebra.

The book starts with a discussion of the axioms of algebra, and then rings, domains of integrity and fields are introduced and the existence of quotient-fields is proved for all domains of integrity. In the third chapter the foundations of group theory are laid down; then follows an investigation of the axioms of divisibility, Euclid's algorithm, and unique decomposition in prime factors. Furthermore the classes of remainders are analysed and the abstract fields are classified according to their *characteristic* as done by Steinitz.

The next chapters in volume I are not very different from the representation of these matters in ordinary textbooks. The only difference is that the coefficients are always elements of an arbitrary abstract field, and that the fields of characteristic $p>0$ in certain cases may cause exceptions. The headings are: Transcendental adjunctions (that is, fields of rational functions of one or more variables), symmetric functions, linear equations, divisibility of polynomials in fields and domains of integrity. An interesting paragraph deals with the determination of the prime-function decomposition in a finite number of operations. The method applied is a direct generalization of Weber's method for algebraic fields.

The existence of abstract fields in which a given polynomial is equal to a product of linear factors is then proved; I remark, en passant, that the proof of this theorem is considerably simpler than the proof for the ordinary fundamental theorem of algebra, which states that *every* polynomial with coefficients in the complex field is equal to a product of linear factors in the same field.

All prime-functions can be divided into two classes: An irreducible polynomial belongs to the first class, when all its roots are different; if not, it belongs to the second class. A field in which all polynomials are of the first class is called *perfect*. The complete Galois theory can be extended to these fields. A field can only then be imperfect, when it has a characteristic $p>0$ and there exists an element a such that $\sqrt[p]{a}$ is not contained in the field.

The last chapters of volume I deal with equations of third and fourth degree and other special equations. In volume II the ordinary fundamental theorem of algebra is proved. Then the Galois theory for all polynomials of the first class is very extensively developed and additional considerations solve the difficulties for polynomials of the second class. This volume contains various subjects new to textbooks. I will only mention the mutual reduction of polynomials and the determination of equations without *affect.* In an addendum Krull studies the Galois theory of algebraic fields of infinite rank, generalized Abelian groups, and their application to the theory of matrices and elementary divisors.

The author assumes that the book should be used for a first textbook in advanced algebra. It seems to me, however, that such a book can only be appreciated by students who already have a rather thorough knowledge of algebra; for a beginner in the subject many of the considerations might look like unnecessary complications and formalities. In general, an axiomatic theory very often has a depressing influence on a reader who is not familiar with the general plan of the building he is going to cooperate on. By taking this attitude the author could have omitted some of the more elementary parts of the book and saved space for other items like, for example, divisibility and the theory of rings. This is, however, no serious reproach to this remarkable book.

OYSTEIN ORE

Mathematical and Physical Papers. By Sir Joseph Larmor. 2 volumes. Cambridge, University Press, 1929. Vol. 1, xii+679 pp., vol. 2, xxxii+831 pp.

The purely mathematical papers form but a small part of the author's work; they were written mostly in the eighties and deal with the geometry of quadric surfaces, transformations of integrals, and the formal aspects of various methods in mathematical physics.

Foremost among the author's contributions to physics are the three, now classical, papers on *A dynamical theory of the electric and luminiferous medium.* Adopting for the electric energy function the form first used by MacCullagh in optics, the author develops the most complete ether theory of electromagnetism ever devised, and correlates it with thermodynamics, kinetic gas theory and other branches of theoretical physics. Much space is given to the theory of the electron, and there are many results in common with the investigations pursued by Lorentz about the same time by different methods. One of the best known of Larmor's results, which has recently acquired added significance in quantum mechanics, is the "Larmor precession" of the orbit of an electron in a constant magnetic field.

Next in importance to the electromagnetic papers are those on general thermodynamics and the theory of gases. There are also a number of papers on geophysical questions, and various reports and addresses.

Larmor handles his mathematics with elegance, but the outstanding features of his work are the clarity and power of his physical reasoning, and the definiteness and comparative simplicity of his mechanical models. A valuable feature of the present edition is the extensive series of historical and critical