ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THIS SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume.* Cross-references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

290. Dr. S. S. Cairns: *The cellular division and approximation of regular spreads.*

An i-cell (Veblen, *Analysis Situs*, The Cambridge Colloquium, Part II, pp. 73–74), $(i, = 1, 2, 3, \cdots, n)$, which is definable by a correspondence of class C' is called *regular*. An *i-spread* is a set of points each of which has for a neighborhood on the set an i-cell or an i-cell plus one of its bounding $(i-1)$-cells. The latter sort of points constitute the boundary of the spread. If (a) the i-cells of this definition are all regular, and (b) the boundary consists of a number of $(i-1)$-circuits (Veblen, loc. cit.) made up of regular $(i-1)$-cells, then the i-spread is called *completely regular*. A *regular i-spread* is one which possesses these properties save on certain $(i-1)$-circuits made up of regular $(i-1)$-cells. It is shown in this paper that a region of euclidean 3-space bounded by a number of regular 2-spreads without boundary is a complex (Veblen, loc. cit.); and that this theorem can be extended to any number of dimensions if the bounding spreads are completely regular. It is shown also that for any regular 2-spread in euclidean 3-space there exists an approximating completely regular 2-spread such that (a) the approximating spread coincides with the given spread save at points arbitrarily near the irregularities of the latter; and (b) a continuous one-to-one correspondence can be set up between the two spreads such that the distance between any pair of corresponding points is arbitrarily small. (Received June 10, 1930.)

291. Professor D. R. Davis: *On minimizing certain types of definite integrals.*

Two types of definite integrals of the form

$$I_1 = \int_{x_1}^{x_2} z^n(1+y'^2+z'^2)^m dx, \quad I_2 = \int_{x_1}^{x_2} (y^2+z^2)^n (1+y'^2+z'^2)^m dx$$

are considered. In each case the expression for the difference between the value of the integral taken along an extremal arc and the value along a comparison curve is given in terms of a certain quadratic form. Hence, by means of known properties of n-ary quadratic forms sufficient conditions for the existence of an extremum for I_1, and also for I_2, are obtained. (Received May 17, 1930.)

* See pp. 1–2 and p. 45 of the January issue.

292. Dr. J. M. Feld: *The expansion of analytic functions in the form* $b_0+\sum_1^\infty b_n a_n z^n/(1-a_n z^n)$.

This paper shows that given any set of constants b_n with the condition that for every n, $|b_{n+1}| \geqq |b_n| \geqq 1$, any function $f(z)$ analytic in the neighborhood of the origin can be uniquely represented in the neighborhood of the origin as a uniformly convergent series of the form $b_0+\sum_1^\infty b_n a_n z^n/(1-a_n z^n)$. From this result it follows that if $f(0)=1$, $f(z)$ can be represented as an absolutely convergent infinite product $\prod_{n=1}^\infty(1-a_n z^n)^{-b_n/n}$ in some neighborhood of the origin. This is a generalization of a theorem by J. F. Ritt, soon to appear in the Mathematische Zeitschrift. (Received May 17, 1930.)

293. Mr. L. S. Kennison: *Linear functional equations in the composite range of a function and n independant variables.*

A systematic theory is developed from the start for the system

$$y(x)=\bar{y}(x)+\int_a^b K(x,\ s).\bar{y}(s)ds+\sum_{j=1}^n K_j(x)\bar{y}^j,$$

$$y^i=\bar{y}^i+\int_a^b K^i(s).\bar{y}(s).ds+\sum_{j=1}^n K_j{}^i\bar{y}^j.$$

This includes a study of the Fréchet differential of the bordered Fredholm determinant and other functionals of similar type. Direct methods are used, and explicit formulas, suitable for future applications, are given. All such transformations whose bordered Fredholm determinants do not vanish are shown to form a group. (Received May 21, 1930.)

294. Professor L. H. McFarlan: *The problem of Lagrange of the calculus of variations for which the integrand contains the coordinates of the end points.*

This paper treats the problem of determining a set of functions $y_1(x), \cdots, y_n(x)$ which give a minimum value to an integral whose integrand contains $x, y_1, \cdots, y_n, y_1', \cdots, y_n'$ and the coordinates of the end points $x_1, y_{11}, \cdots, y_{n1}, x_2, y_{12}, \cdots, y_{n2}$. The arguments of the integrand also satisfy a system of $m<n$ differential equations. A transversality condition is obtained from the first variation. From the second variation, a boundary value problem is set up in terms of which one may determine the sign of the second variation and from which one may also derive a condition analogous to the Jacobi condition in the more simple problem of the calculus of variations. (Received May 19, 1930.)

295. Professor A. D. Michal and Mr. L. S. Kennison, *Quadratic functional forms in a composite range.*

This paper develops a theory of quadratic functional forms in a function $y(x)$ and n independent variables $y^1, y^2, \cdots, y^n$. This study, in particular, includes as a special case a unified theory of the classical quadratic form theory in n variables and the recent work on quadratic functional forms. An application, of some importance, is made to functional forms in $y(x)$ alone, continuous of order one, with continuous coefficients. One of the theorems proved is that the theory of such forms can be reduced to that of forms, continuous of order zero, in a function $\phi(x)$ and a variable Y with continuous coefficients. (Received May 23, 1930.)

296. Dr. W. E. Milne: *On the maximum absolute value of the derivative of* $e^{-x^2}P_n(x)$.

A remarkable theorem due to S. Bernstein asserts that if L is the maximum absolute value of an arbitrary polynomial $P_n(x)$ of degree n in the interval (a, b) then the maximum absolute value of the derivative $P_n'(x)$ does not exceed $nL\,[(b-x)(x-a)]^{-1/2}$ on (a, b). A related theorem for trigonometric sums states that if L is the maximum of the absolute value of a trigonometric sum of order n, then the maximum absolute value of its derivative does not exceed nL. In this paper a similar result is obtained for the function $e^{-x^2}P_n(x)$, where $P_n(x)$ is an arbitrary polynomial of degree n. It is shown that if L is the maximum absolute value of $e^{-x^2}P_n(x)$ in the interval $-\infty<x<\infty$ then the maximum absolute value of the derivative is less than $n^{1/2}L[1.0951+O(n^{-1})]$ in the infinite interval. (Received May 16, 1930.)

297. Professor R. E. Moritz: *On doubly-depleted Fourier's series of the types* $\sum_1^\infty f(n)\cos(nx)$ *and* $\sum_1^\infty f(n)\sin(nx)$, *where* $f(n)$ *is a homogeneous function of* n.

A series of the types indicated in which n is limited to positive integers prime to a given integer P is designated as a depleted Fourier's series as distinguished from a complete Fourier's series in which no terms are missing. The series is said to be singly-depleted if P is a prime number, doubly-depleted if P consists of the product of two prime factors, multiply-depleted if P consists of the product of more than two factors. The paper deals with doubly-depleted series of the types mentioned. Multiply-depleted Fourier's series will be treated in a subsequent paper. (Received May 15, 1930.)

298. Mr. J. B. Small: *A center-surface for the ellipsoid.*

Cayley discussed the center-surfaces for an ellipsoid generated by the points P_1 and P_2 on the inward normal at distances from the surface equal to the principal radii of curvature C_1 and C_2. The author determines the surface generated by a like point P, whose distance from the surface is the mean proportional of C_1 and C_2. Various interesting singularities are found, and the surface is drawn for several shapes of ellipsoids. (Received May 24, 1930.)

299. Miss Florence Swanson: *Curved wires of minimum attraction for a given particle.*

Given a particle P and two equidistant points A, B, the author studies the shape of a fine curved wire (in the plane) through A and B which shall exert upon P the minimum attraction. The differential equation arising from Euler's condition is replaced by two others, of the first order, relating r and θ to the polar directional angle ψ as an auxiliary variable. By graphical charts and step-by-step integration a series of extremals are determined, and the curves drawn. Sufficiency tests are presented. (Received May 24, 1930.)

300. Professor J. W. Campbell: *On the determination of stringing tensions for transmission lines and cables.*

In this paper there is given a means of calculating a temperature-tension relation for any given span. The problem is treated under three cases: (1) points of support at the same level; (2) points of support at different levels; (3) composite spans which consist of several individual spans to be strung simultaneously. Hyperbolic methods are used throughout, and the details are arranged to facilitate computations. In the concluding paragraph it is shown that while the tension in a hanging cable is not uniform over the length, it is unnecessary to consider this variation in tension when finding a temperature-tension relation for the span. (Received June 20, 1930.)

301. Professor J. V. Uspensky: *Incomplete numerical functions.*

In a paper *On Gierster's Class-Number Relations* the author has given a very general identity similar to those published by Liouville. In the present paper he gives another general identity of the same kind and devises several curious formulas involving incomplete numerical functions. (Received June 20, 1930.)

302. Professor J. V. Uspensky: *Ch. Jordan's series for probability.*

In an article published in the Bulletin de la Societé Mathématique de France, Ch. Jordan gave a very remarkable series capable of representing a given infinite series of numbers and made use of this series to represent the probabilities in the Bernoullian series of trials. Ch. Jordan's development is purely formal and does not contain a method to establish the convergence of his series. In the present paper the question concerning the convergence of Ch. Jordan's series is solved under very general assumptions and moreover, the practically important question of the rapidity of the convergence of those series is elucidated. (Received June 20, 1930.)

303. Professor A. D. Michal: *Theorems concerning functional geodesic coordinates.*

The theory and application of the subject of n-dimensional geodesic coordinates of order r has been given by the author in two previous papers (Abstracts 36-3-159 and 36-3-209). The present paper is concerned with corresponding theories in an affinely connected function space. Certain applications are given in the next paper. (Received July 1, 1930.)

304. Professor A. D. Michal: *Scalar functional invariants of a continuous one-parameter family of functional vectors.*

This paper is concerned with applications of some of the theorems of the author's preceding paper, *Theorems concerning functional geodesic coordinates*, to the calculation of absolute scalar functional invariants of a continuous one-parameter family of contravariant (covariant) functional vectors in a function space with a symmetric affine connection. In particular, the analogues of Ricci's coefficients of rotation are introduced. (Received July 1, 1930.)

305. Professor A. D. Michal: *An introduction to differential geometries of composite spaces.* Preliminary report.

This paper marks the beginnings of a theory of tensors and various types of differential geometries in the composite spaces whose points are determined by a function $y(s)$, $a \leqq s \leqq b$, and n independent variables $x_1, x_2, \cdots, x_n$. Theories of "linear connections," "covariant differentiation," and "infinitesimal parallelism" are developed. As explicit special cases of the general subject of this paper, one can cite two noteworthy instances: (1) the theory of n-dimensional affinely connected manifolds; (2) the author's studies on differential geometries of the space of the continuous functions $y(s)$ (American Journal, 1928; Proceedings of the National Academy, 1930). (Received July 1, 1930.)

306. Mr. L. S. Kennison: *Projective transformations in function space.*

An application of work on linear functional equations in a composite range is made to projective transformations in function space. All the results given by L. L. Dines (Transactions of this Society, vol. 20 (1919), p. 45) are obtained somewhat more simply. It is proved for the first time that the one-parameter family of finite transformations generated by an infinitesimal projective functional transformation is a one-parameter continuous group of projective functional transformations. (Received July 1, 1930.)

307. Professor W. L. Ayres: *On the regular points of a continuum.*

In this paper a study is made of the points of finite order (in the sense of Urysohn and Menger) in any continuum. It is shown that for $n>2$ the set of all points of order n forms a zero-dimensional set. This is a corollary of a more general result that the set of all points p such that $n \leqq$ order of $p \leqq 2n-3$ is a zero-dimensional set. It is proved that the set of points of order 2 consists of a countable number of arcs plus a zero-dimensional set, and each arc-segment is an open subset of the continuum. As it is already known that the points of order 1 are at most zero-dimensional, this completes the knowledge of the structure of the points of order n for any finite integer n. (Received May 21, 1930.)

308. Mr. K. E. Bisshopp: *Abstract defining relations for the simple group of order* 5616.

The simple group of order 5616 may be generated by an operator T of order 2 and an S of order 3. The product of these two must be of order 13, and they must satisfy one of two sets of conditions. The particular set of relations satisfied by such an S and T is also satisfied by S^{-1} and T. (Received July 7, 1930.)

309. Professor John Eiesland: *On a certain class of ruled V_{n-1}^{n-1} in S_n.* Second paper.

This paper is the continuation of one which will appear in the next issue of the Palermo Rendiconti. The singular loci of the general V_{n-1}^{n-1} in even and odd space have been enumerated. In even space it is found that, besides the *regular loci* such as $(n/2)$-fold points, $((n-2)/2)$-fold 2-flats, $\cdots$, r-fold $(n-2r)$-flats, there exist also double, triple, $\cdots$ r-fold spreads of dimension $d=n-2r+2-2^{r-1}$. These are the *accessory singular loci.* All the singular loci are of even dimension. In odd space, we have a similar result, but the singular loci are now of odd dimension. A few interesting properties of these accessory loci have been considered. The V_5^5 in S_6 and the V_6^6 in S_7 have been treated somewhat in detail. The Grassmannian coordinates have been used throughout. (Received July 5, 1930.)

310. Professor O. E. Glenn: *Preliminary report on a new method in the theory of central orbits.*

Invariant equations, which we derive from integral invariants, represent transcendental curves each having a segment in practical coincidence with a given segment of the orbit. The accuracy of the coincidence increases with a parameter. The result is an advantageous viewpoint, in problems relating to an orbit, where a segment is the primary object of study, for the reason that the transcendental curves are simpler to deal with analytically than the orbital equation itself. An instance of such a problem is the central potential which can be determined from a segment. (Received June 26, 1930.)

311. Professor J. I. Hutchinson: *Note on the number of linearly independent Dirichlet series that satisfy certain functional equations.*

In a paper published in the Transactions of this Society, volume 31, pages 324–344, which considers four different types of functional equations satisfied by ordinary Dirichlet series, the author was unable to give a rigorous determination of the number of linearly independent functions (of the form considered) in two out of the three cases that arise. The present note fills this gap by determining in a very simple manner the multiplicity of the roots of the characteristic determinant on which the solution of the problem depends. This is done by observing that the sum of these roots is equal to the sum of the coefficients occurring in the main diagonal of the determinant, and this sum can be evaluated in all cases by means of Gauss's sums. The results confirm the conjectures advanced in the paper referred to. (Received June 24, 1930.)

312. Professor M. H. Ingraham: *An elementary theorem on matrices.*

By noting the equivalence of certain matrix problems to problems in the elementary theory of numbers, a generalization to matrices of the theorem concerning the existence of a polynomial of degree $n-1$ having given values for n distinct values of the variable is found. Among other results the following is noted. If the minimum equations of n finite square matrices m_1 to m_n with elements in a field F are relatively prime, then, for any set of n polynomials $p_1, \cdots, p_m$ in F, a polynomial f may be found such that $f(m_i) = h_i(m_i)$, $i = 1, \cdots, n$. The solution is essentially unique. (Received May 5, 1930.)

313. Professor Edward Kasner: *Element transformations and isogonal series.*

An isogonal series is defined by the author to mean ∞^1 elements (x, y, p) which make a constant angle α with the curve obtained by joining the points of the elements. If $\alpha = 0$ the series becomes a union or curve. For an arbitrary element transformation it is shown that there exist ∞^4 isogonal series which are converted into isogonal series. This quadruple family includes the triple family discussed in the author's previous paper (Abstract 36–5–239), namely the curves turned into isogonal series; also of course the ∞^2 curves turned into curves (American Journal, 1910). For any given value of α, the base curves of the ∞^3 series have properties analogous to those for the special case $\alpha = 0$. But for a given value of the angle β of the transformed series the situation is much more complicated. (Received June 12, 1930.)

314. Dr. N. H. McCoy (National Research Fellow): *On some general commutation formulas.*

Let $x_i (i = 1, 2, \cdots, n)$ be any class of elements satisfying the ordinary laws of algebra except that multiplication is non-commutative. In this paper we obtain two identities for the commutator, $fg - gf$, where f is an arbitrary defined function of the x's and g is of the form $\sum a_{i_1, i_2, \cdots, i_n} x_1^{i_1} x_2^{i_2} \cdots x_n^{i_n}$. The identities are given in terms of expressions of the form $\phi x_i - x_i \phi$, where ϕ is a function of the x's. From these identities we can obtain some general commutation formulas for functions of p and q, these being subject to the relation $pq - qp = c$, which is characteristic of quantum mechanics. The formulas obtained here are in a different form from those given in a previous paper. It is found, for example, that if f and g are arbitrary polynomials in p and q, then $fg - gf = \sum c^s \sum_{i+j=s} [(-1)^i/(i!j!)]\ [(\partial^s g/\partial p^i \partial q^j) - (\partial^s f/\partial q^i \partial p^j)]$. Corresponding results are obtained for the case of n pairs of quantum variables satisfying similar relations. Some further applications are made to functions of three variables, α, β, γ, which satisfy the following commutation rules: $\alpha\beta - \beta\alpha = c\gamma$, $\gamma\alpha - \alpha\gamma = c\beta$, $\beta\gamma - \gamma\beta = c\alpha$. (Received July 5, 1930.)

315. Professor G. Y. Rainich: *On interlinkings.*

Gauss gave a formula for the number of interlinkings of two curves in the form of a double integral. This formula is proved in the present paper by first showing that the integral is a special case of the Kronecker-Picard integral and, therefore, its value is always an integer; it follows that a continuous deformation of the curves during which they do not acquire common points does not change the value of the integral; this permits us to reduce the general case to standard cases in which the calculation is very easy, and so to prove Gauss's formula. Next the formula is generalized for two closed subspaces of a space whose number of dimensions exceeds by one the sum of the numbers of dimensions of the subspaces. (Received July 7, 1930.)

316. Professor C. N. Reynolds: *Closed circuits upon polyhedra.*

This paper deals with the conditions under which it is possible to assign a cyclic order to the regions (or vertices) of a polyhedron such that any two consecutive elements are incident with one and only one edge of the polyhedron. An earlier result due to Mr. Whitney (Abstract 36-3-187) is extended, and the relations between this problem and the four-color problem are considered. The methods used are suggested by the theory of those maps which are irreducible with respect to the four-color problem. (Received July 7, 1930.)