Les Problèmes des Isopérimètres et des Isépiphanes. By T. Bonnesen. Paris, Gauthier-Villars, 1929. iv+175 pp.

Here is the latest volume of the Collection Borel. Readers having as little Greek as the reviewer will appreciate the kindness of the author in translating the last word of the title on line 4 of the introduction; it means solids of given surface.

After introductory chapters on elementary maxima and minima, and Jensen's theory of convex functions, the author passes to the main subject matter, viz. the classical extremal properties of the circle and the sphere, and their generalizations, notably the Minkowski inequalities for the mixed volume of three convex solids. The very elegant methods of treatment are original with the author and consist in geometrical constructions, often surprisingly simple, on polygons and polyhedra, supplemented by elementary algebraic calculations and limit operations. The main difficulty in this subject consists in determining the cases where an inequality of the type considered reduces to an equality. This difficulty the author overcomes very neatly by setting up stronger inequalities, from which the answer may be read off at once. For instance, he replaces the ordinary isoperimetric inequality in the plane, $L^2/(4\pi)-S\geqq 0$, by $L^2/(4\pi)-S\geqq(\pi/4)(R-r)^2$, where L is the length of a convex curve, S the area it encloses, and R and r the radii of the circumscribed and inscribed circles. Now it is seen at once that in the ordinary inequality, equality takes place only when $R=r$, that is, for the circle.

The last chapter deals with the extension of the methods used to certain more general problems in the calculus of variations. There is also a fairly complete bibliography.

T. H. GRONWALL

Géometrie sur les Surfaces et les Variétés Algébriques. By S. Lefschetz. Mémorial des Sciences Mathématiques, fascicule XL. Paris, Gauthier-Villars, 1929. 66 pp.

This pamphlet is a fairly complete account of that advanced part of the theory of algebraic surfaces and varieties which deals with the transcendental and geometric properties of a variety which are invariant under birational transformations. The timeliness of such a report is unquestionable. Since the appearance of Enriques' and Castelnuovo's article in the Encyclopedia (1914) the theory has been enriched by many new important results, and also a new development of the theory along the lines of analysis situs has taken place (Lefschetz). This development has thrown new light upon the stupendous progress already achieved by the theory thanks to the investigations of the Italian geometers Castelnuovo, Enriques, Severi, etc., and the French analysts Humbert, Picard, Poincaré. The pamphlet thus serves a useful purpose in furnishing a survey of the present state of the theory, and also gives the exact measure of the distance travelled by the theory since 1914.

The exposition centers around three main topics: the transcendental theory of the integrals attached to an algebraic variety V, the geometric theory of linear and continuous systems on V, and the analysis situs of V. Each of these topics leads in its own way to the very heart of the theory of algebraic varieties. The material is so arranged as to bring forth the far reaching connections between these three topics, which thus appear only as different aspects of what one may call the theory of (numerical and functional) invariants of an algebraic variety. Although the geometric, analytic and transcendental points of view are freely allowed to have full play, the stress is laid upon the analysis situs applications, which, we think, is fully justified by the fact that even specialists in algebraic geometry are not yet sufficiently familiar with the topological aspects and methods of the theory.

The author does not confine himself to mere exposition of known results. When necessary, unsolved questions are raised and freely discussed, and this valuable feature of the pamphlet makes it something more than a résumé At the end a rather inspiring list of possible new researches to be pursued is given.

Some details may be criticized. For instance, Theorem 27 on page 24 is given somewhat casually, without any accompanying comment, which is against the historic and intrinsic importance which this theorem had in the development of the theory of algebraic surfaces. In the discussion of the problem of the existence of double integrals of the 1st kind without periods (p. 28) it would have been advisable to mention explicitly the closely connected problem of the existence of simple integrals of the so-called semiexact differentials of the first kind, introduced by Severi.

There are a few misprints which may mislead the reader. On p. 15, XIV, "$k<d-2$," "Γ_{k-2}" and in formula (10) "C^{k+1}" should be respectively: "$k\leqq d-2$," "Γ_k" and "C^{d-k-1}". On p. 16 fourth line from bottom and on p. 17, XX, "C^{d-k}" and "C^{d-k+1}" should be "C^k" and "C^{k+1}", respectively.

OSCAR ZARISKI

Cours de Mécanique Professé à l'École Supérieure des Mines. By P. Lévy Paris, Gauthier-Villars, 1928. 8+205 pp.

This book was prepared by Professor Lévy, of l'École Polytechnique, on the occasion of his assignment to l'École Supérieure des Mines. The point of view differs from that of such standard treatises as Appell's in that more attention is given to mechanical and geometrical considerations. For example in studying the dynamics of a particle, the author feels that it is more important for the student to understand and appreciate the form of the trajectory than the analytic formulas defining it.

The technical applications are almost entirely confined to kinematics. After the first chapter on theoretical kinematics there are two comparatively long chapters on applications to machinery. Then follow eight chapters giving the customary treatment of the dynamics of a particle and of rigid bodies. The next chapter considers the equilibrium of flexible systems and includes a breaf treatment of graphical statics.

It is implied in the preface that this volume is intended to furnish a foundation for the oral instruction of the course and hence it does not contain exercises for the student. Some material not covered in the oral instruction is included in the book with the hope that the curiosity of the better students will be aroused. For this reason the volume closes with a supplementary chapter on the theory of relativity.

W. R. LONGLEY