The Elementary Theory of Tensors. By Tracy Yerkes Thomas. New York, McGraw-Hill, 1931. 10+122 pp.

This text presents the elements of geometry and mechanics in tensor notation. It will in consequence be useful either as a basis for a modernized course in vector analysis or as a reference for the undergraduate who delves into popular Einsteiniana and asks his teacher "What are tensors?".

The argument is well and carefully thought out. In particular, the fundamental equations of euclidean and kinematical transformations are derived by methods not readily available elsewhere. It is perhaps a regrettable consequence of the chòice of applications that, with the exception of the vector product, all the physical or geometrical examples of tensors mentioned are first-order tensors, that is, vectors. Besides a few harmless misprints the reviewer noted only two mistakes. The footnote on page 68 implies that every rigid displacement of the plane is a rotation with finite center, although the close reader will recall the mention in the text of the case in which the center recedes to infinity. On page 37 we read "No legal standard of length has been adopted by the United States and in the absence of such a standard the British system of measurement has come into practical use."

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Vorlesungen über die Singulären Moduln und die Komplexe Multiplikation der Elliptischen Funktionen. Part II (Teubner's Sammlung, Band XLI, 2). By R. Fueter. Leipzig, Teubner, 1927. 143–358 pp.

This is the second and concluding volume on the subject by Professor Fueter, the first of which appeared in 1924 and was reviewed by Dresden in the American Mathematical Monthly (vol. 32 (1925), pp. 474–476). The reader is urged to consult the Dresden review for orientation. Let us recall that in Part I the author has developed, compactly, the theory of the modular group and associated functions, and the algebraic-arithmetic theory of quadratic number fields, including ideal-theory and the many concepts stemming from this important branch of mathematics.

In Part II Professor Fueter enters the deeper realms of his subject, giving particularly the important extensions since Weber's *Algebra*, vol. III. While most of the book is arithmetic and algebraic in character, analytic methods are by no means eschewed. Thus, in the course of the volume, there is a much-trodden path leading from the elliptic function addition theorems, and the author, travel far as he might in the higher theory of number fields, never loses contact with these fruitful formulas. In addition, we find on a few occasions that the zeta-functions of Dirichlet and Dedekind are utilized to establish a theorem neatly. The author has given in his preface a two-fold explanation of the advantages of function-theoretic methods; algebraists will be interested in the points that he makes.

The present volume is a continuation of the other both in chapter numbering and paging. Thus we begin with Chapter VI, on the factorization of prime ideals. It should be recalled that the fundamental number fields in this theory are the quadratic imaginary number fields $k(\sqrt{m})$ (or simply k). With respect to k one considers larger containing fields K, and in particular such fields K as have, in k, an abelian group. A field of this character is a relative-abelian field (with respect to k). Important examples are *class fields* and *ray class fields*, introduced in vol. I. The present chapter is devoted to the proof of a group of theorems on the factorization of prime ideals in both class fields and ray class fields. We find here the first use (in the book) of the zeta-functions; and by their means it is established that the ray class equation is irreducible. On the other hand, the class equation is shown to be factorable upon adjoining to k a certain number of square roots. A similar theorem pertains to the ring class equation.

Chapter VII studies in detail the general ray class field and its relative discriminant. First there is a careful treatment of those ray class fields whose leaders are multiples of the ideal (4), after which the general case is handled. The theorems of this chapter relate to the composition of ray class fields, the nature of the prime ideals that are contained in the relative discriminant of $K(4)$, and the *Relativdifferente* of a ray class field. In particular, if f is an ideal of k, then the ray class field determined by choosing f as a leader is relatively Abelian with respect to k. Finally, for the general ray class field there is a fundamental theorem (pages 262–263) which is an excellent summing up (space and technicality of language do not permit quoting); a summing that brings one out of the maze of details and proofs onto a summit from which one can make a survey of the next problem.

That problem is the proof of *completeness*, which, in the words of the author, has been the goal of the whole book and is the crown of the entire theory. What is this theorem of completeness, which holds, and rightly, so high a place in the author's esteem? It can be put in different forms; one of the most striking is this. *Every abelian equation in a quadratic imaginary field $k(\sqrt{m})$ can be resolved by means of singular moduli and singular elliptic functions.* (These singular quantities are defined in volume I.) The first proof of this fine result appears in the present volume.

There is a Chapter IX on the computation of the singular values of a certain modular function and of associated equations. A good part of this chapter is function-theoretic in character. The problem of computation is by no means a simple one, as any reader will see.

So much for the mathematical content of the book; and that stands high indeed. What of the personal content: the style, the ease with which the material can be grasped? It has been intimated that the book is written very compactly; and so it is. There are, for example, as many as 254 theorems in the space of 305 pages (8 chapters), and most of the proofs are by no means trivial. This suggests that perhaps a student would be carried from page to page, from theorem to theorem, so intent on mastering proofs as to miss the important stations on the journey. And indeed, a first study of the volume is likely to be so characterized. But, we can acknowledge, a second reading brings topographic "relief": the high points *do* stand out, and the journey through the book, (to the original contents of which Professor Fueter has notably contributed), is a fascinating one. Certainly, experienced algebraists will find the book profitable and important, as a noteworthy extension of knowledge in this field. To those with but a modicum of knowledge of the subject matter, the book will still be fascinating; but, let us admit it, for such the road will not be a royal one.

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