Consider a surface $F_n$ of order $n$ in $r$-space. Let it be the complete intersection of $q \leq r - 2$ varieties $V_{k_1}^{n_1}, V_{k_2}^{n_2}, \ldots, V_{k_q}^{n_q}$ of orders $n_1, n_2, \ldots, n_q$ and of dimensions $k_1, k_2, \ldots, k_q$, respectively, where
\begin{align*}
3 &\leq k_1, k_2, \ldots, k_q \leq r - 2, \\
k_1 + k_2 + \cdots + k_q &= r(q - 1) + 2.
\end{align*}

Project $F_n$ on an $S_3$. The projection $F'^n$ has a number of characteristics of which we note the following six: $n$, its order; $a$, the order of its tangent cone; $b$, the order of its double curve; $j$, the number of its pinch-points; $t$, the number of its triple points; and $m$, its class. If we project $F_n$ on an $S_3$, the projection has a finite number, $d$, of improper double points. We shall call these seven characteristics, of which $n, a, t, m$ are often regarded as essential, the characteristics of $F_n$, and they are known to satisfy the following relations:*\begin{align*}
a + 2b &= n(n - 1), \quad j + 2d = n(n - 1) - a, \\
j &= \frac{1}{3} \left[ a(3n - 4) - n(n - 1)(n - 2) + 6t - 2m \right], \\
d &= \frac{1}{3} \left[ n(n - 1)(n + 2) - 3an - 6t + 2m \right].
\end{align*}

For $r = 5, q = 3, k_1 = k_2 = k_3 = 4$, $F_n$ is the intersection of three hypersurfaces in $S_3$. Formulas for its characteristics are known † and they are symmetric functions of the orders of the hypersurfaces. In this note we present analogous formulas for the same characteristics of $F_n$ for $r$ general and for $q \leq r - 2$. As the method of obtaining these formulas is familiar and has been applied by the writer time and again to similar enumerative problems, ‡ we shall here omit all demonstration.

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† B. C. Wong, *On surfaces in spaces of four and five dimensions*, this Bulletin, vol. 36 (1930), pp. 681–686. Opportunity is here taken to correct an error in the formula for $T''$ on page 685 of this paper. The formula should read $T'' = \frac{1}{3} \mu \nu (\lambda - 1)(\mu - 1)(\nu - 1)(\mu \nu + \nu \lambda + \lambda \mu - 2\lambda - 2\mu - 2\nu)$.
‡ B. C. Wong, loc. cit., and also the paper *On the number of apparent double points of r-space curves*, this Bulletin, vol. 37 (1931), pp. 421–423.
If \( q = r - 2 \), and, from (A), \( k_1 = k_2 = \cdots = k_{r-2} = r - 1 \), \( F^n \) is the complete intersection of \( r-2 \) hypersurfaces in \( S_r \). The formulas for its characteristics are

\[
\begin{align*}
n &= n_1 n_2 \cdots n_{r-2}, \\
a &= n(\sum n_i - r + 2), \\
b &= \frac{1}{2}n(n - \sum n_i + r - 3), \\
d &= \frac{1}{2}n\left[n - \sum n_i n_j + (r - 4) \sum n_i - \frac{1}{2}(r - 3)(r - 4)\right], \\
j &= n\left[\sum n_i n_j - (r - 3) \sum n_i + \frac{1}{2}(r - 2)(r - 3)\right], \\
t &= \frac{1}{6}n\left[n(n - 3) \sum n_i + 3(r - 3)(n - 2) \sum n_i \right. \\
&\quad\left. + 2(\sum n_i^2 + 3 \sum n_i n_j) + (r - 3)(3r - 8)\right], \\
m &= n\left[\sum(n_i - 1)^2 + \sum(n_i - 1)(n_j - 1)\right], \\
\end{align*}
\]

(i \( \neq j \)).

Now if \( q \leq r - 2 \), one or more of the \( k \)'s will be less than \( r - 1 \). Let the \( i \)th variety \( V_i^m \) be intersected by a general \( S_{r+2-k_i} \) in a surface \( F^m \). We assume known the characteristics \( a_i, b_i, t_i \) of \( F^m \) besides \( n_i \). The characteristics of \( F^m \) are given by the following formulas which are functions of \( n_i \) and \( q \), and also of \( a_i, b_i \) and \( t_i \):

\[
\begin{align*}
n &= n_1 n_2 \cdots n_q, \\
a &= n(\sum n_i - q) - 2n \sum b_i/n_i = n \sum a_i/n_i, \\
b &= \frac{1}{2}n(n - \sum n_i + q - 1) + \sum b_i/n_i = \frac{1}{2}n(n - 1) - \frac{1}{2}n \sum a_i/n_i, \\
d &= \frac{1}{2}n\left[n - \sum n_i n_j + (q - 2) \sum n_i - \frac{1}{2}(q - 1)(q - 2)\right] \\
&\quad+ n \sum_i(\sum n_i - q + 1)b_i/n_i - 2n \sum b_i b_j/n_i n_j, \\
j &= n\left[\sum n_i n_j - (q - 1) \sum n_i + \frac{1}{2}q(q - 1)\right] \\
&\quad- 2n \sum_i(\sum n_i - q)b_i/n_i + 4n \sum b_i b_j/n_i n_j, \\
t &= \frac{1}{6}n\left[n(n - 3) \sum n_i + 3(q - 1)(n - 2) \sum n_i \right. \\
&\quad+ 2(\sum n_i^2 + 3 \sum n_i n_j) + (q - 1)(3q - 2)\right] + n \sum t_i/n_i \\
&\quad+ n \sum_i[n - 2 \sum n_i - n_i + 2(q - 1)]b_i/n_i + 4n \sum b_i b_j/n_i n_j, \\
m &= n\left[\sum(n_i - 1)^2 + \sum(n_i - 1)(n_j - 1)\right] + 3n \sum t_i/n_i \\
&\quad- n \sum(2 \sum n_i + 3n_i - 2q + 2)b_i/n_i + 4n \sum b_i b_j/n_i n_j, \\
\end{align*}
\]

(i \( \neq j \)).

All the formulas of each of these two sets satisfy relations (B).

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